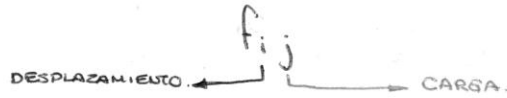


# METODO DE RIGIDEZ

FLEXIBILIDAD DESPLAZAMIENTO QUE PRODUCE UNA CARGA UNITARIA.



● ES CONOCIDO COMO EL METODO DE LAS CARGAS.

$$\{\Delta\} = [F] \{\phi\}$$

EXPRESION MATEMATICA.

RIGIDEZ ES LA CARGA QUE PRODUCE UN DESPLAZAMIENTO UNITARIO.



● ES CONOCIDO COMO EL METODO DE LOS DESPLAZAMIENTOS.

$$\{\phi\} = [K] \{\Delta\}$$

EXPRE. MATEMATICA.

Donde:  $\{\phi\}$  = ES UN VECTOR DE CARGAS QUE ACTUA SOBRE LOS NUDOS.

$\{\Delta\}$  = VECTOR DESPLAZAMIENTO DE NUDOS.

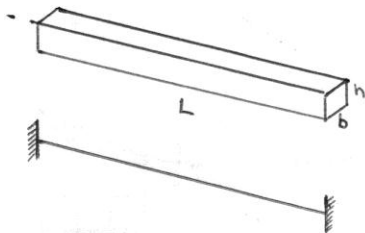
$[K]$  = MATRIZ DE RIGIDEZ DE LA ESTRUCTURA.

$f$  (# DE NUDOS)

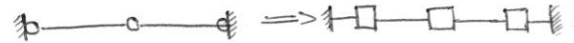
PARA "n" NUDOS.

$$\begin{Bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_n \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} & \dots & K_{1n} \\ K_{21} & K_{22} & & \\ & & \ddots & \\ K_{n1} & & & K_{nn} \end{bmatrix} \begin{Bmatrix} \Delta_1 \\ \Delta_2 \\ \vdots \\ \Delta_n \end{Bmatrix}$$

NUDO. PARTE DIFERENCIAL DE UNA ESTRUCTURA.



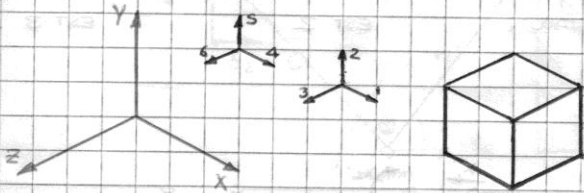
$$b, h \ll L$$



SE RECOMIENDA CONSIDERAR NUDOS EN:

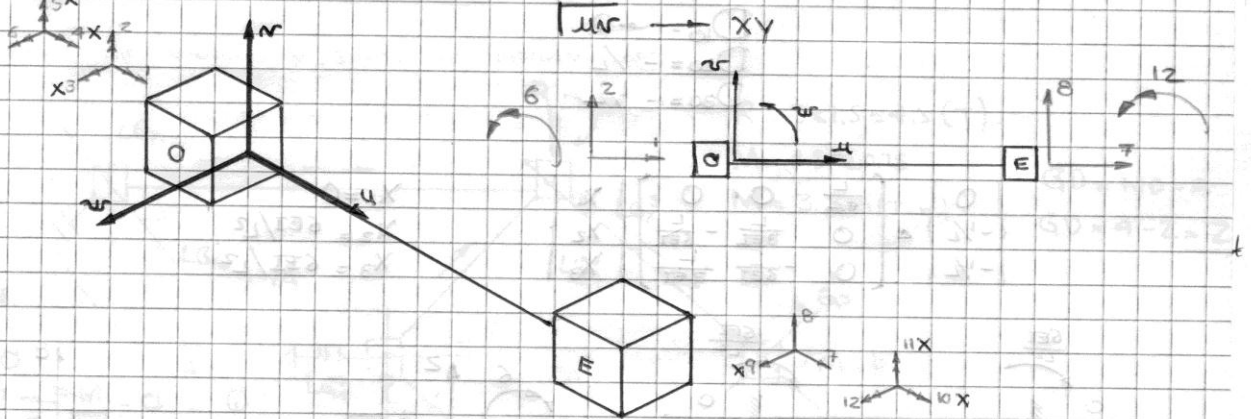
- APOYOS
- CAMBIOS DE DIRECCION
- EN EL CRUCE DE DOS O MAS ELEMENTOS
- CAMBIO DE MATERIAL
- CAMBIO DE SECCION
- DONDE SE QUIERE CONOCER EL DESPLAZAMIENTO

GRADO DE LIBERTAD SON LOS DESPLAZAMIENTOS QUE PUEDE TENER UN NUDO.

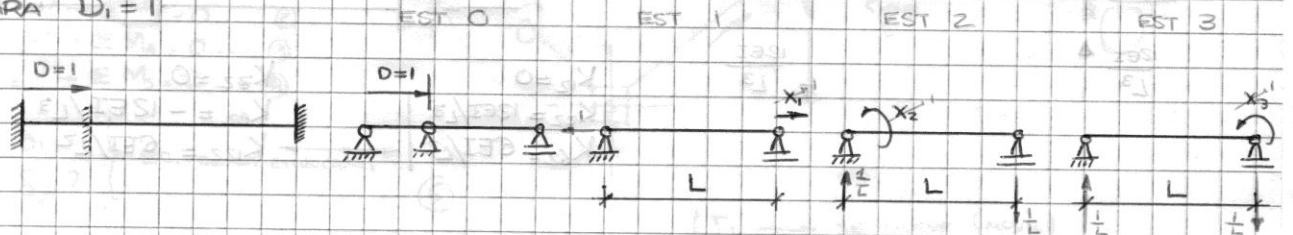


EN EL ESPACIO UN NUDO TIENE 6 GL

BARRAS



PARA  $D_1 = 1$



$$D_{10} = 1$$

$$N_2 = 1$$

$$N_2 = 0$$

$$N_2 = 0$$

$$D_{20} = 0$$

$$Q_2 = 0$$

$$Q_2 = \frac{1}{L}$$

$$Q_2 = \frac{1}{L}$$

$$D_{30} = 0$$

$$M_2 = 0$$

$$M_2 = \frac{X}{L} - 1$$

$$M_2 = \frac{X}{L}$$

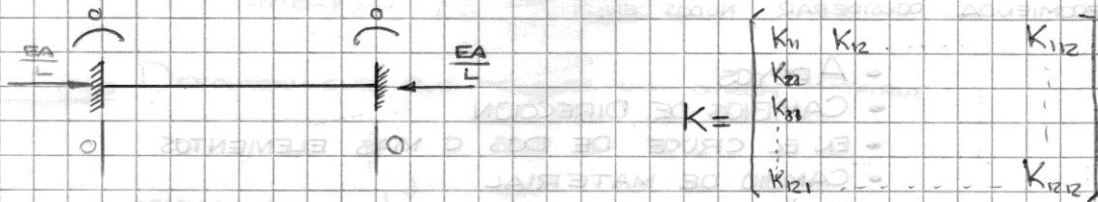
$$- \begin{Bmatrix} D_{10} \\ D_{20} \\ D_{30} \end{Bmatrix} = \begin{Bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{Bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix}$$

$$X_1 = - \frac{EA}{L}$$

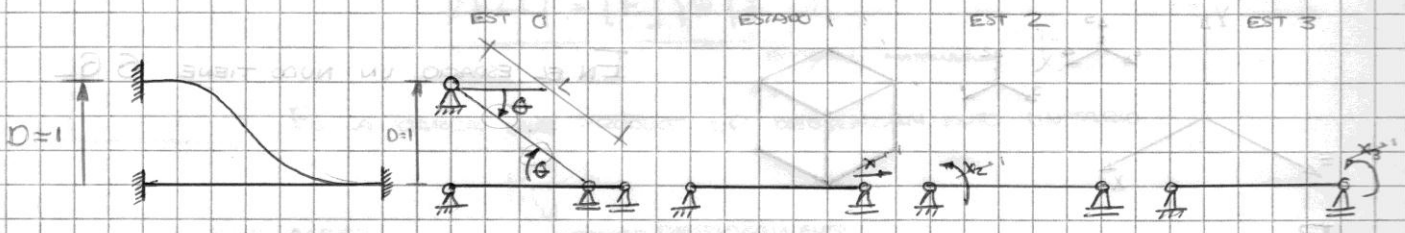
$$X_2 = 0$$

$$X_3 = 0$$

$$\begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} \frac{L}{EA} & 0 & 0 \\ 0 & \frac{L}{3EI} & -\frac{L}{6EI} \\ 0 & -\frac{L}{6EI} & \frac{L}{3EI} \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix}$$



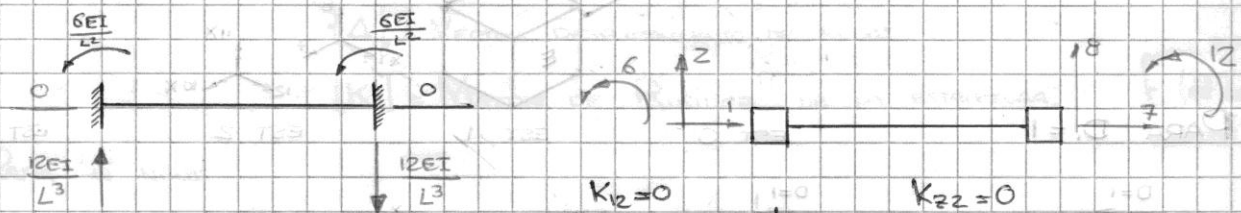
$K_{11} = \frac{EA}{L}$      $K_{41} = 0$      $K_{61} = -\frac{EA}{L}$   
 $K_{21} = 0$      $K_{51} = 0$   
 $K_{31} = 0$      $K_{61} = 0$



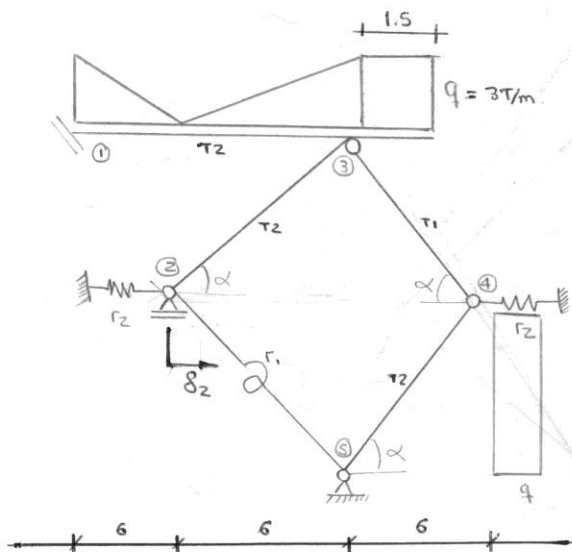
$D_{10} = 0$   
 $D_{20} = -1/L$   
 $D_{30} = -1/L$

$$\begin{bmatrix} 0 \\ -1/L \\ -1/L \end{bmatrix} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{L}{3EI} & -\frac{L}{6EI} \\ 0 & -\frac{L}{6EI} & \frac{L}{3EI} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

$X_1 = 0$   
 $X_2 = 6EI/L^2$   
 $X_3 = 6EI/L^2$



$K_{12} = 0$      $K_{22} = 0$   
 $K_{22} = 12EI/L^3$      $K_{62} = -12EI/L^3$   
 $K_{62} = 6EI/L^2$      $K_{122} = 6EI/L^2$



$T1 \rightarrow 20 \times 45$   
 $T2 \rightarrow 20 \times 40$   
 $r_1 = SE3 T-m/red.$   
 $r_2 = 4E3 T-m$   
 $E = 2.1E5 Kg/cm^2$

Sol.

① CARACTERÍSTICAS GEOMÉTRICAS.

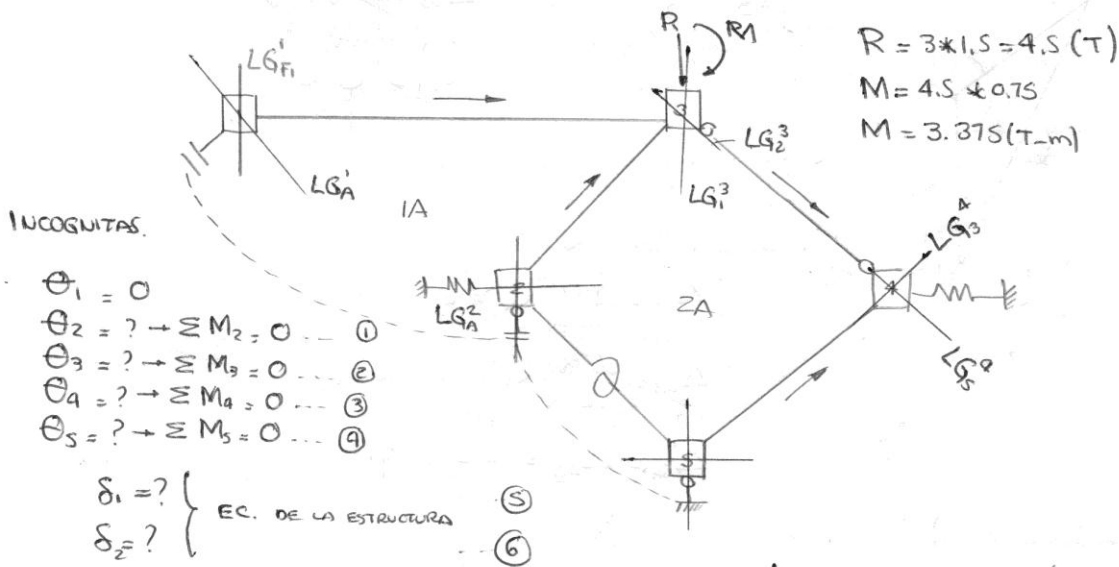
$L_{23} = L_{34} = \sqrt{3^2 + 6^2} = 6.708 (m)$

$\alpha = \tan^{-1}(3/6) = 26.565^\circ$

$EI_1 = \frac{0.2 \times 0.45^3}{12} \times 2.1E6 = 3184.375 (T-m^2)$

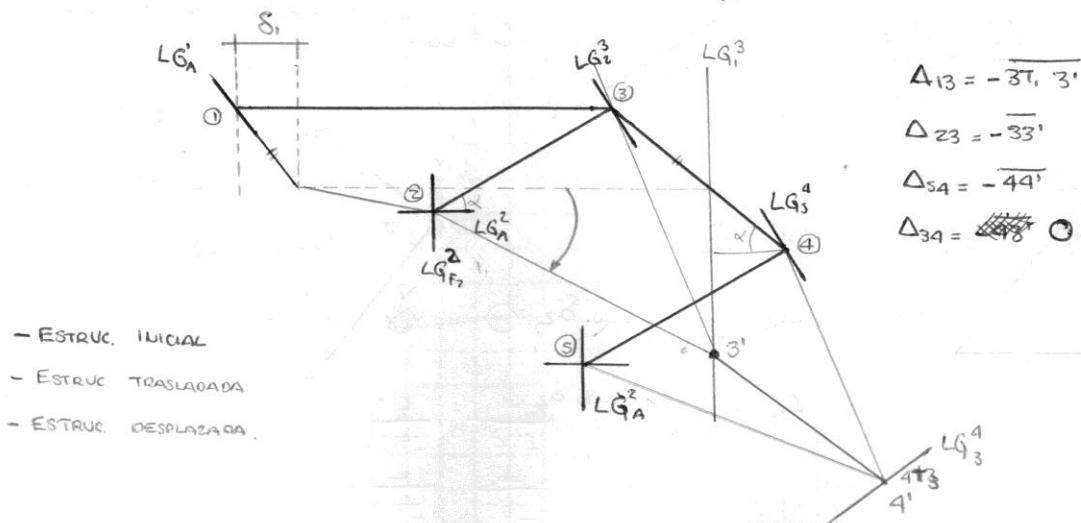
$EI_2 = \frac{0.2 \times 0.4^3}{12} \times 2.1E6 = 2240.000 (T-m^2)$

② CONFIGURACION DE VINCULOS Y GRADO DE DESPLAZABILIDAD.

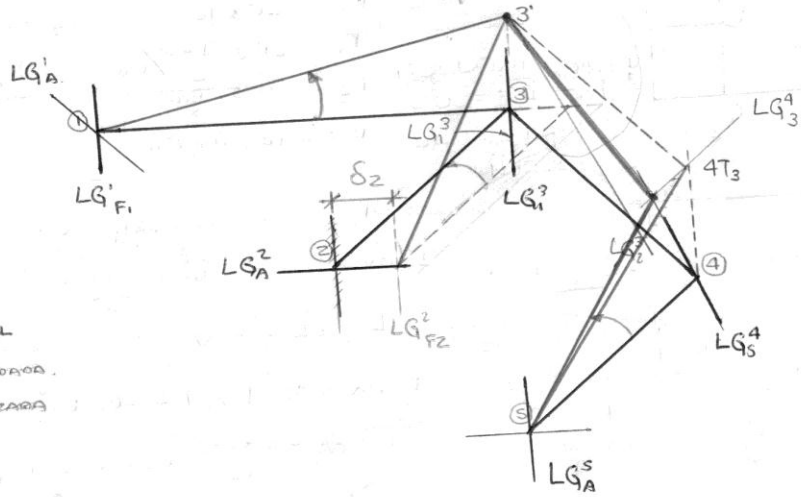


③ ESTRUCTURA DESPLAZADA. Y CALCULO DE DESPLAZAMIENTOS.

$F_1 \rightarrow$  SE MUEVE (MOVIL)  
 $F_2 \rightarrow$  PERMANECE FIJO (NO SE MUEVE)



ESTRUCTURA DESPLAZADA PARA  $\delta_2$



$$\Delta_{13} = \overline{33'}$$

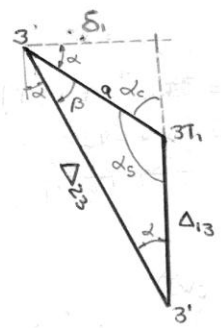
$$\Delta_{23} = \overline{3T_2 3'}$$

$$\Delta_{34} = \overline{44'}$$

$$\Delta_{34} = -\overline{4T_3 4'}$$

- EST. INICIAL
- EST. TRASLAPADA
- EST. DESPLAZADA

CALCULO DE DESNIVELES PARA  $\delta_1$



$\alpha = 26.565^\circ$

$\alpha_c = 90 - \alpha = 63.435^\circ$

$\alpha_s = 180 - \alpha_c = 116.565^\circ$

$\beta = 90 - 2\alpha = 36.870^\circ$

$\cos \alpha = \frac{\delta_1}{a} \rightarrow a = 1.118 \delta_1$

$$\Delta_{13} = -3T_1 3'$$

$$\Delta_{23} = -\overline{33'}$$

$$\Delta_{34} = -\overline{44'}$$

$$\Delta_{34} = 0$$

Por la ley de senos.

$$\Delta_{13} = \frac{\sin \beta}{\sin \alpha} a = 1.500 \delta_1$$

$$\Delta_{23} = \frac{\sin \alpha_s}{\sin \alpha} a = 2.236 \delta_1$$

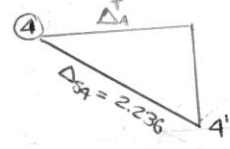
$$\Delta_{34} = \Delta_{23} = 2.236 \delta_1$$

DESPLAZAMIENTO EN LOS RESORTES.

NUDO ② FIJO

$$\Delta_2^T = 0$$

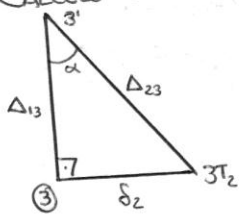
NUDO ④



~~$$\Delta_{34} = \Delta_{23} = 2.236 \delta_1$$~~

$$\Delta_4^T = 2.236 \delta_1 \sin \alpha = 1.000 \delta_1$$

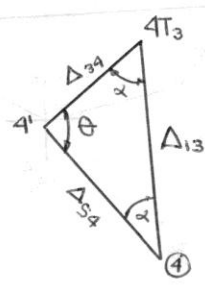
CALCULO DESNIVELES PARA  $\delta_2$



$$\text{tg } \alpha = \frac{\delta_2}{\Delta_{13}}$$

$$\Delta_{13} = \frac{\delta_2}{\text{tg } \alpha} = 2.000 \delta_2$$

$$\Delta_{23} = \frac{\delta_2}{\text{sen } \alpha} = 2.236 \delta_2$$



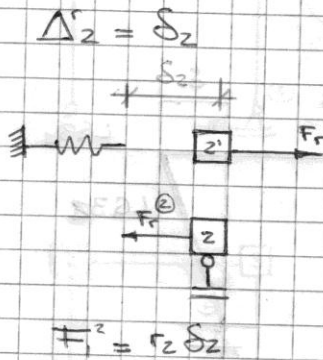
$\theta = 180 - 2\alpha = 126.870^\circ$

$$\Delta_{34} = \frac{\text{sen } \alpha}{\text{sen } \theta} \Delta_{13} = 1.118 \Delta_{13}$$

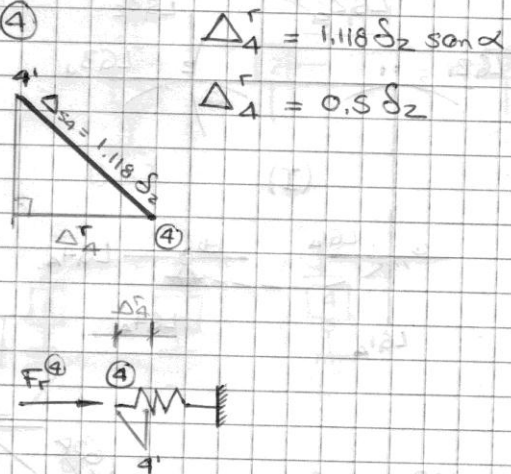
$$\Delta_{34} = \frac{\text{sen } \alpha}{\text{sen } \theta} \Delta_{23} = 1.118 \Delta_{23}$$

DESPLAZAMIENTO EN LOS RESORTES.

NUDO ②



NUDO ④



\* ACORTAMIENTO DEL RESORTE

$\Delta_4^r = \delta_1 - 0.5 \delta_2$



$F_4^r = r_2 ( \delta_1 - 0.5 \delta_2 )$

• RESUMEN DE DESNIVELES TOTALES.

$\Delta_{13} = -1.5 \delta_1 + 2 \delta_2$

$\Delta_{23} = -2.236 \delta_1 + 2.236 \delta_2$

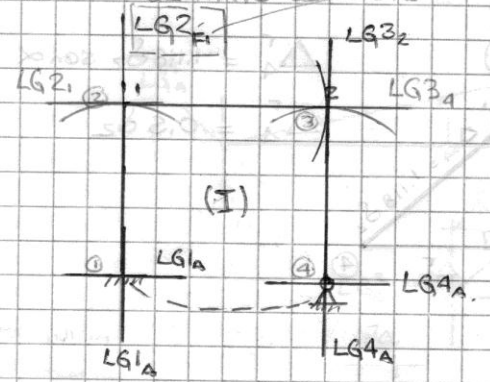
$\Delta_{54} = -2.236 \delta_1 + 1.118 \delta_2$

$\Delta_{34} = 0 \delta_1 - 1.118 \delta_2$

$\Delta_2^r = 0 \delta_1 + \delta_2$

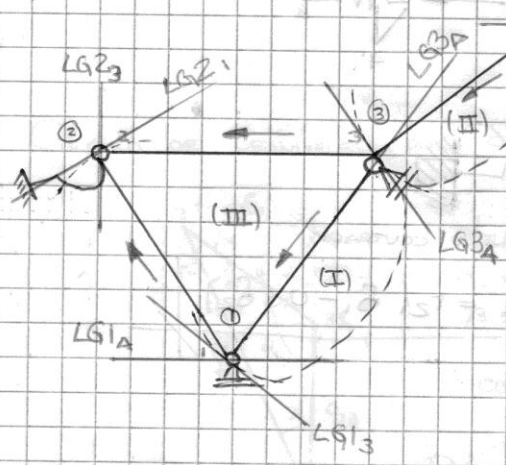
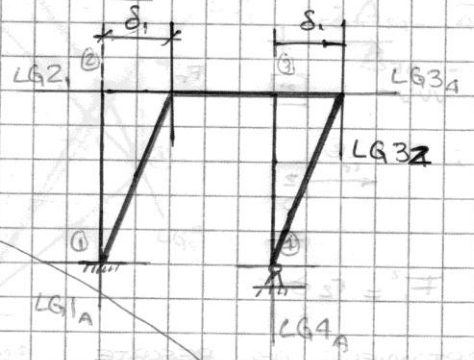
$\Delta_4^r = \delta_1 - 0.5 \delta_2$

ORURO 7 DE MAYO DE 2010



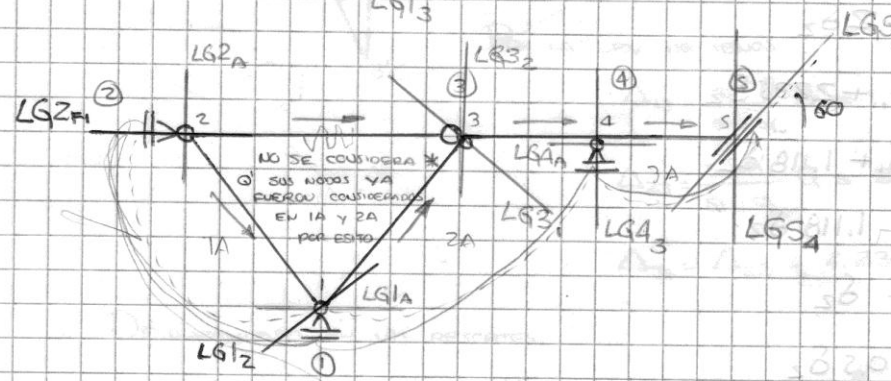
$$GD = ND - A$$

$$GD = 2 - 1 = 1$$



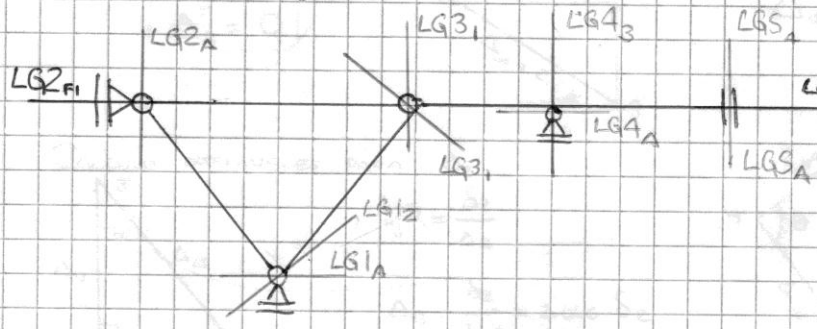
$$GD = ND - A$$

$$= 4 - 3 = 1$$



$$GD = 5 - 3 = 2$$

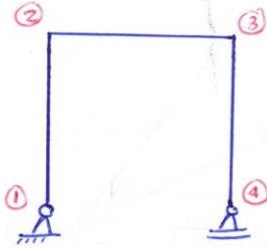
CANBIANDO UN ARCO RELEAMOS



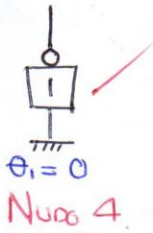
$$GD = ND - A$$

$$GD = 5 - 3 = 2$$

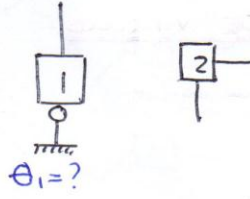
CONFIGURACION DE NUDOS



Nudo 1



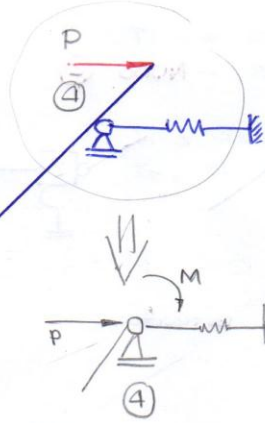
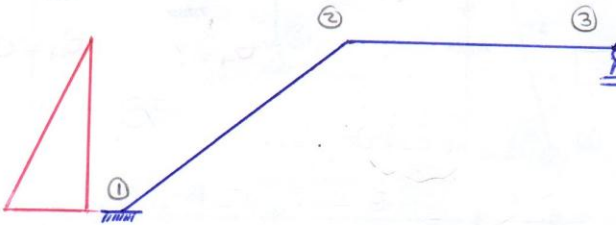
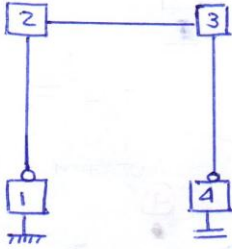
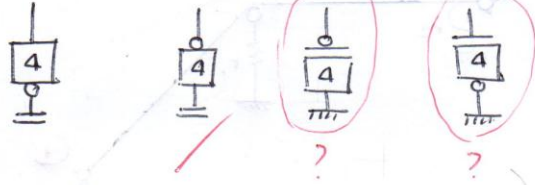
Nudo 2



Nudo 3



Nudo 4



0 and 1

M →

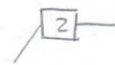
F →

EL RECORTE  
NO TRABAJA

Nudo 1



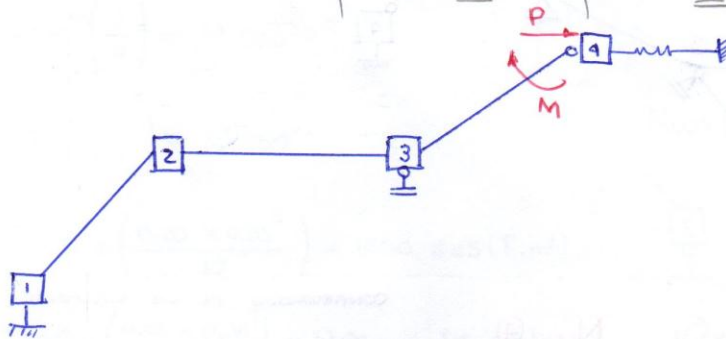
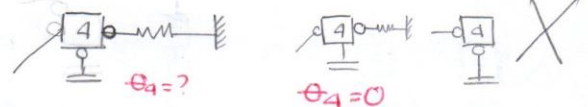
Nudo 2



Nudo 3



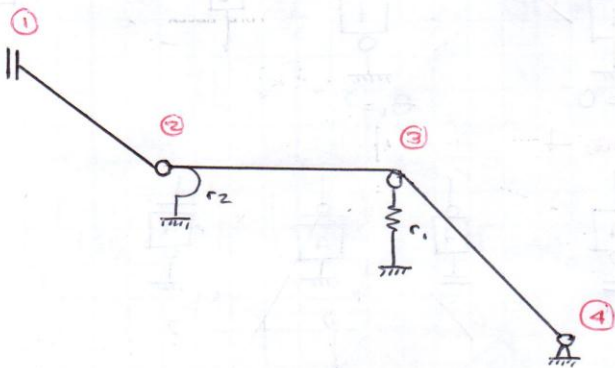
Nudo 4





MAYO 11 DE 2010

CONFIGURACION DE VINCULOS.



Nudo ①



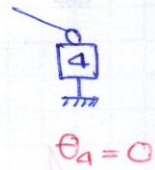
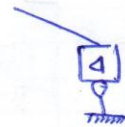
Nudo ②



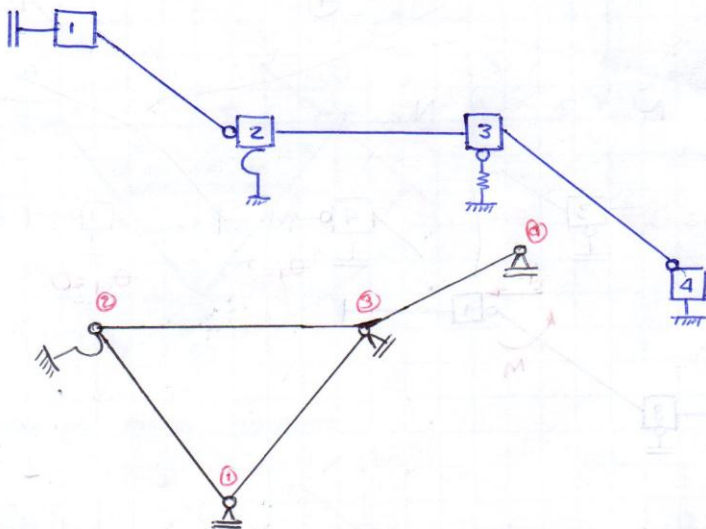
Nudo ③



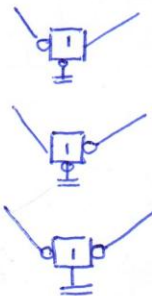
Nudo ④



CONFIGURACION DE LISTRA ESTRUCTURAL.



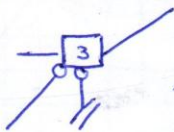
Nudo ①



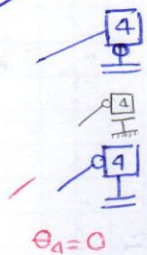
Nudo ②



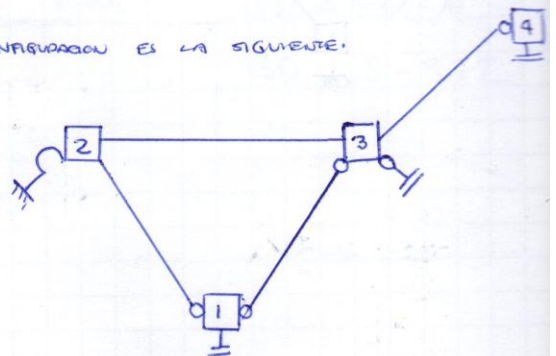
Nudo ③

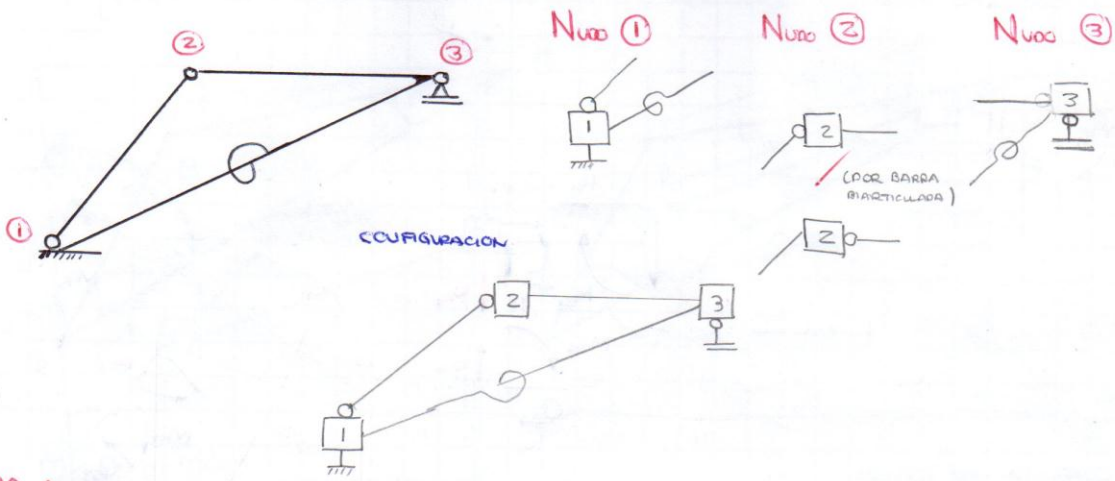


Nudo ④

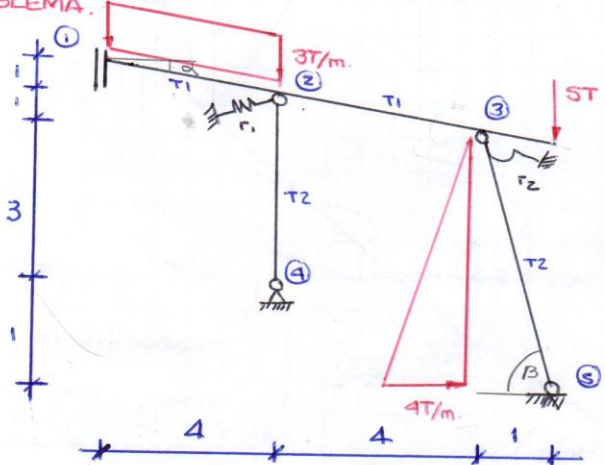


CONFIGURACION ES LA SIGUIENTE.





**PROBLEMA.**



$$\left. \begin{aligned} T1 &\rightarrow 20 \times 35 \\ T2 &\rightarrow 20 \times 30 \end{aligned} \right\} E = 2.1 \text{ E}6 \text{ (Kg/cm}^2\text{)} \\ E = 2.1 \text{ E}6 \text{ (Tn/m}^2\text{)}$$

$$\Gamma_1 = 5 \text{ E}3 \text{ T/m}$$

$$\Gamma_2 = 4 \text{ E}3 \text{ T}\cdot\text{m/rad.}$$

**1) CARACTERISTICAS DE LA ESTRUCTURA.**

$$L_{12} = L_{23} = \sqrt{1^2 + 4^2} = 4.123 \text{ (m)}$$

$$L_{35} = \sqrt{4^2 + 1^2} = 4.123 \text{ (m)}$$

$$\alpha = \text{tag}^{-1}\left(\frac{1}{4}\right) = 14.036^\circ$$

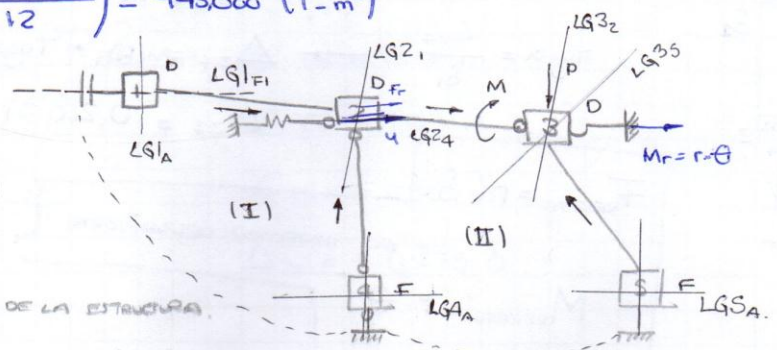
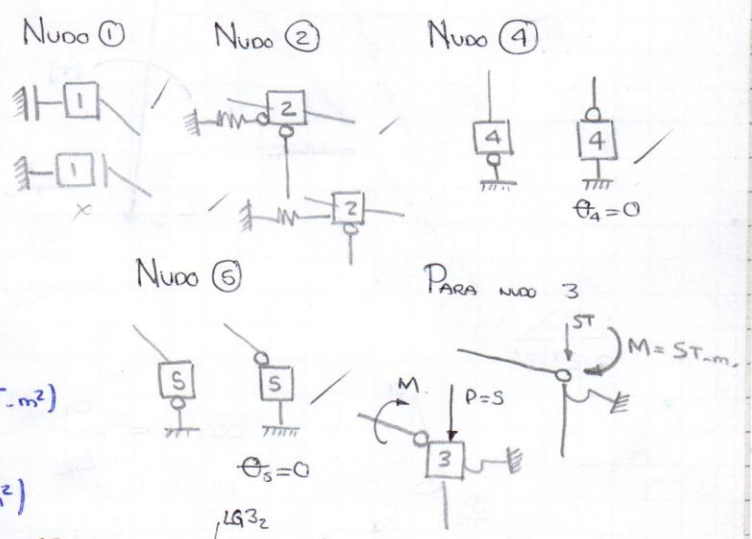
$$\beta = \text{tag}^{-1}\left(\frac{4}{1}\right) = 75.964^\circ$$

$$EI_1 = 2.1 \text{ E}6 \times \left(\frac{0.20 \times 0.35^3}{12}\right) = 1500.625 \text{ (T}\cdot\text{m}^2\text{)}$$

$$EI_2 = 2.1 \text{ E}6 \times \left(\frac{0.20 \times 0.30^3}{12}\right) = 945.000 \text{ (T}\cdot\text{m}^2\text{)}$$

- $\theta_1 = 0$
- $\theta_2 = ? \rightarrow \sum M_2 = 0$
- $\theta_3 = ? \rightarrow \sum M_3 = 0$
- $\theta_4 = 0$
- $\theta_5 = 0$
- $\delta_1 = ? \rightarrow \text{EQUACION DE LA ESTRUCTURA}$

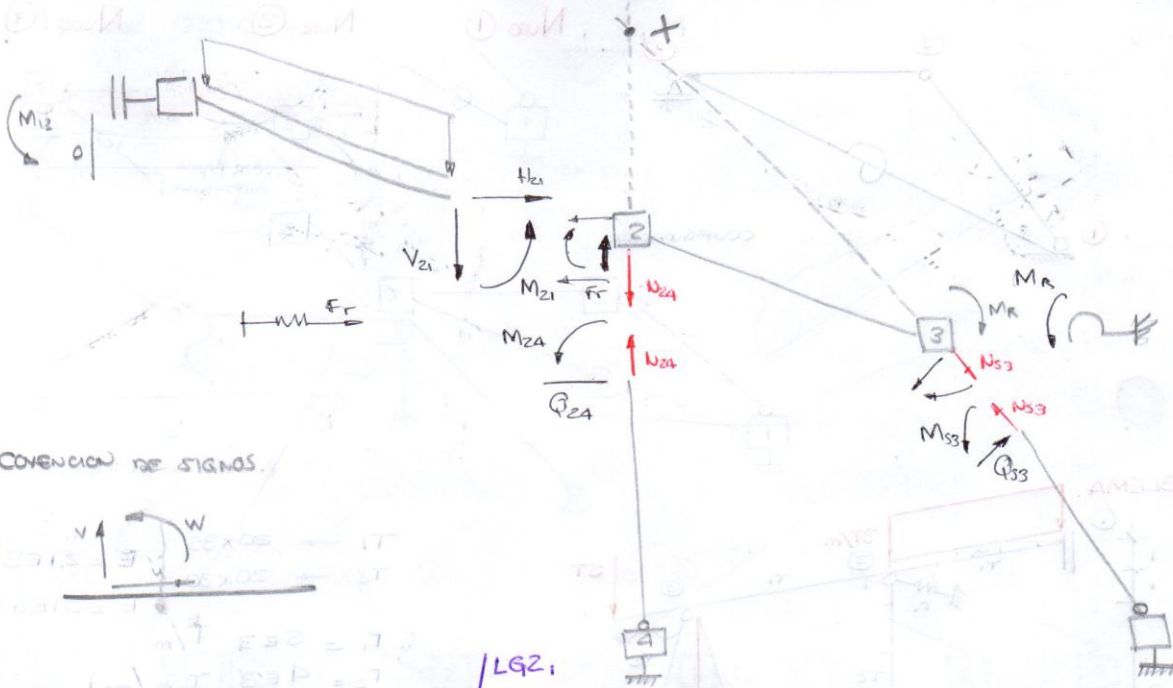
**2) CONFIGURACION DE VINCULOS Y GRADO DE DESPLAZAMIENTO**



$$GD = ND - A$$

$$GD = 3 - 2 = 1$$

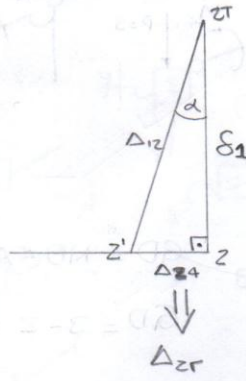
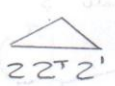
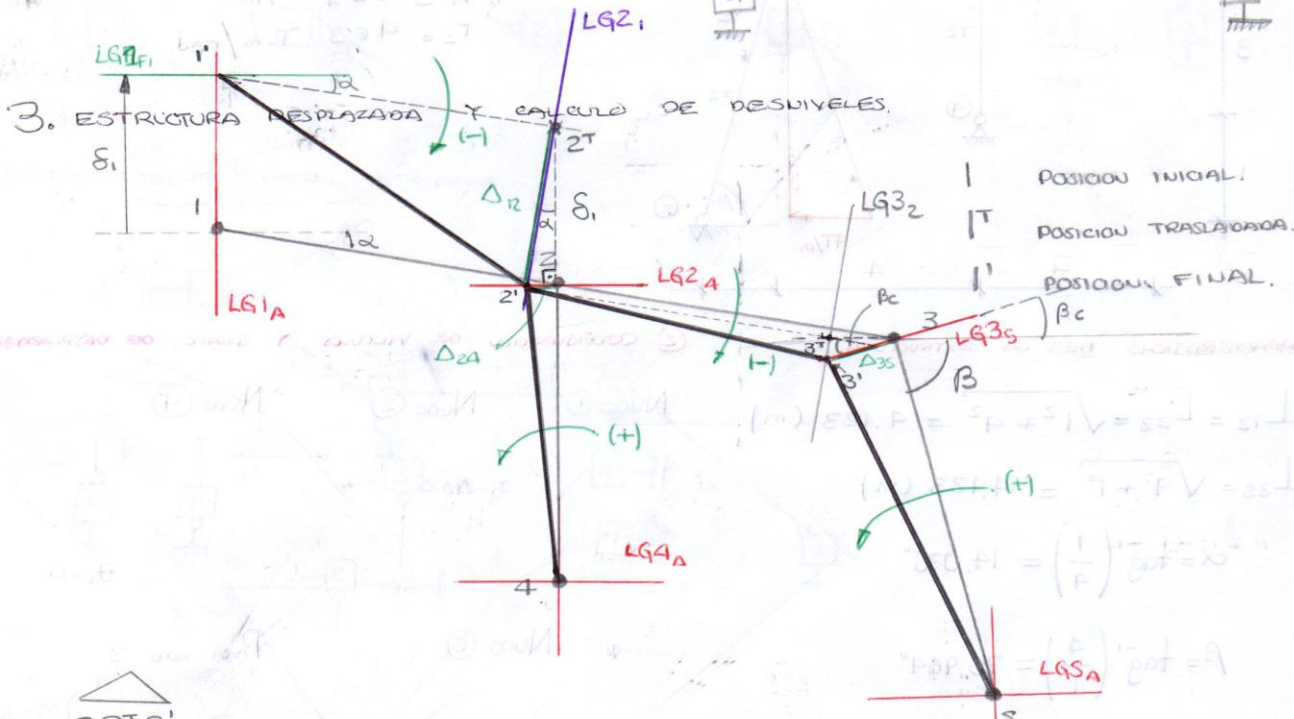
$$F_r = r \cdot \Delta r$$



CONVENCIÓN DE SIGNOS.



3. ESTRUCTURA DESPLAZADA Y CÁLCULO DE DESNIVELES.



$$\cos \alpha = \frac{\delta_1}{\Delta_{12}} \rightarrow \Delta_{12} = \frac{\delta_1}{\cos \alpha}$$

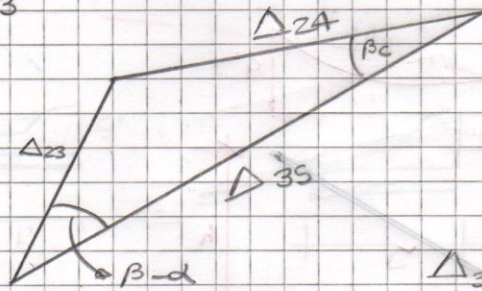
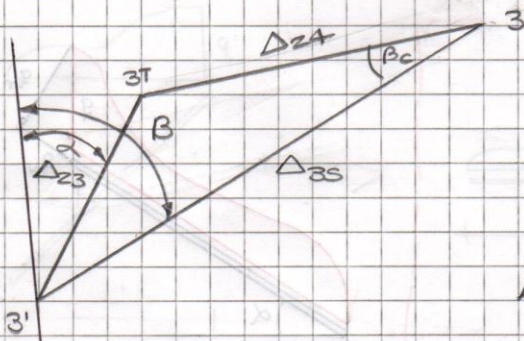
$$\tan \alpha = \frac{\Delta_{24}}{\delta_1} \rightarrow \Delta_{24} = \delta_1 \cdot \tan \alpha$$

$$\Delta_{24} = 0,250 \delta_1$$

$$\Delta_{2r} = 0,250 \delta_1$$

$$\alpha = 14,036^\circ$$

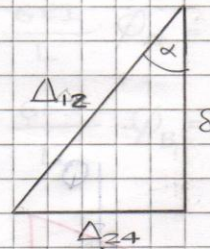
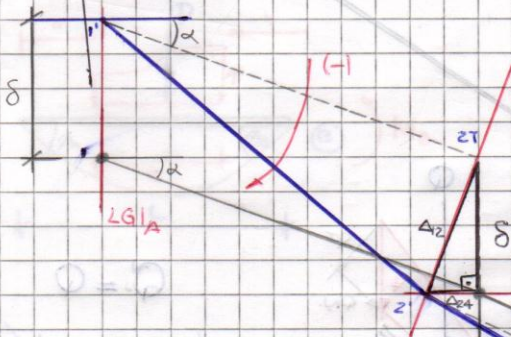
$F_{RESORTE} = r \times \delta$   
 EN DIRECCION DEL RESORTE  
 $M_{RESORTE} = r \cdot \theta$



$$\Delta_{35} = 0,275 \delta$$

$$\Delta_{23} = 0,063 \delta$$

• 18 DE MAYO DE 2010

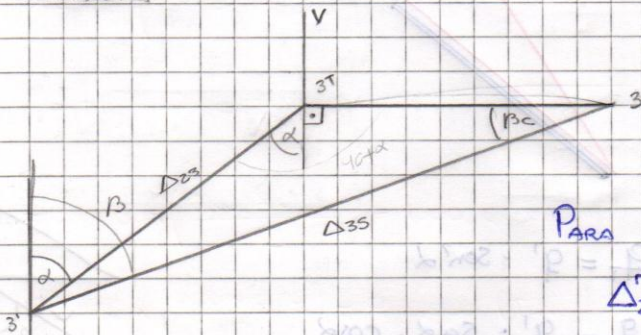


$$\Delta_{12} = 1,031 \delta$$

$$\Delta_{24} = 0,250 \delta$$



OTRA MANERA



PARA EL RESORTE

$$\Delta_2^r = \Delta_{24} = -0,250 \delta$$

RESUMEN DE DESNIVELES.

$$\Delta_{12} = -1,031 \delta$$

$$\Delta_{23} = -0,069 \delta$$

$$\Delta_{35} = +0,275 \delta$$

# METODO ROTACIONES ANGULARES (METODO DE RIGIDEZ)

- Est. ISOSTÁTICAS
- Est. HIPERESTÁTICAS

## 1. BASE FUNDAMENTAL

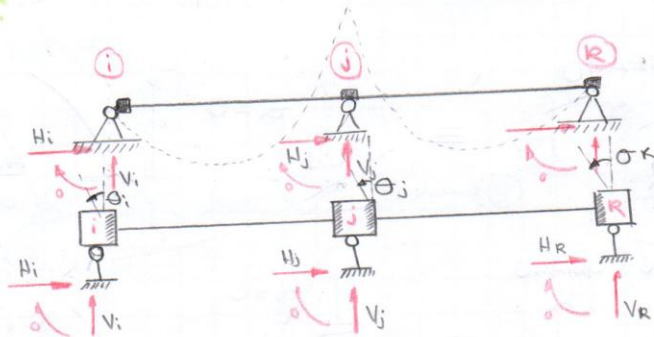
INCOGNITAS SON DESPLAZAMIENTOS (LINEALES Y ROTACIONALES) DE PUNTOS O NUDOS DEFINIDOS POR DISEÑADOR.

### 1.1. CONFIGURACION DE VINCULOS

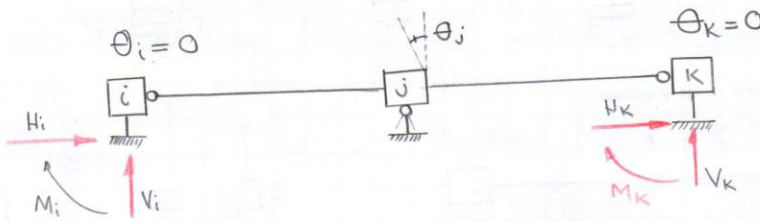
ES EXPRESAR UN NUDO DE FORMA DIFERENCIAL AMPLIADA, CON LOS TIPOS DE LIBERTADES Y

RESTRICCIONES QUE LA CONFORMAN.

EJEMPLO:



3 INCOGNITAS



1 INCOGNITA

DATOS

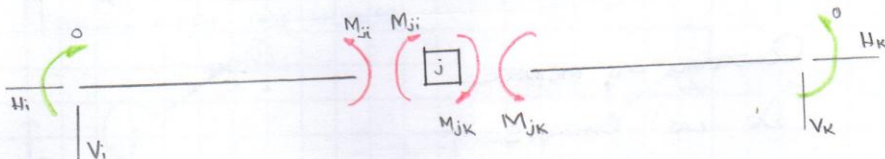
$$\theta_i = 0$$

$$\theta_k = 0$$

INCOGNITAS

$$\theta_j = ?$$

ANALIZANDO NUDO j



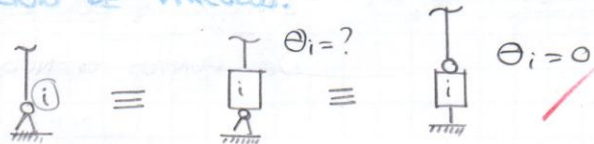
$$\sum M_j = 0 \quad M_{ji} + M_{jk} = 0 \quad \text{--- (1)}$$

$$M_{ji} = f(\theta_j)$$

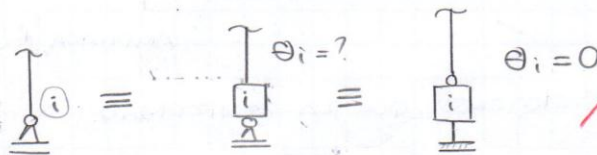
$$M_{jk} = g(\theta_j)$$

## 1.2. TIPOS DE CONFIGURACION DE VINCULOS.

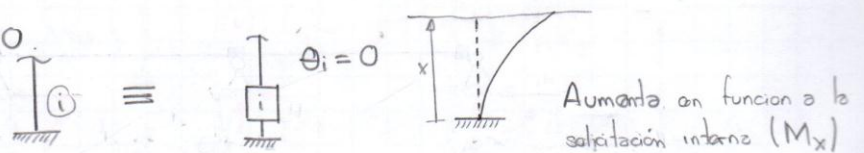
Apoyo Fijo



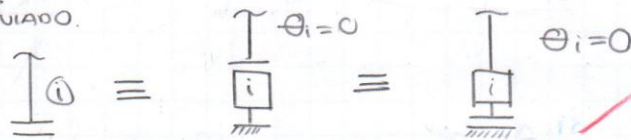
Apoyo MOVIL



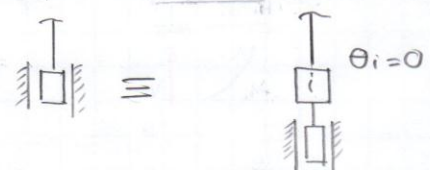
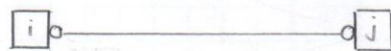
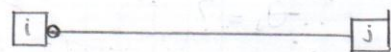
Apoyo Empotrado



Apoyo Empotrado-Guiaado



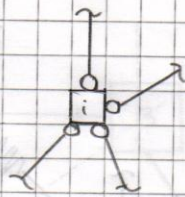
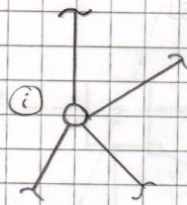
CONFIGURAR DE UNA DE ESTAS 4 FORMAS



CONOCER LAS RIGIDEZES.

DE LAS BARRAS.

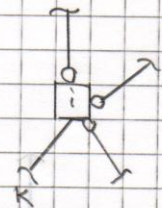
## • UNION ARTICULADA



$$\theta_i = ?$$

$$\sum M_i = 0$$

$$0 = 0$$

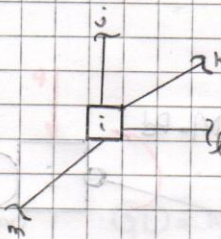
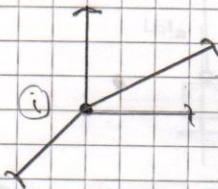


$$\theta_i = ?$$

$$\sum M_i = 0$$

$$M_{iK} = 0$$

## • UNION EMPOTRADA

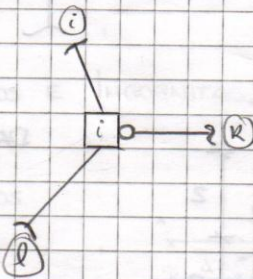
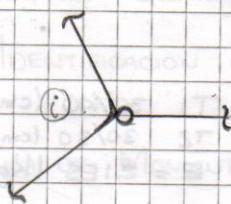


$$\theta_i = ?$$

$$\sum M_i = 0$$

$$M_{ij} + M_{iK} + M_{il} + M_{im} = 0$$

## • UNION MIXTA

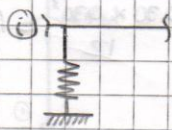


$$\theta_i = ?$$

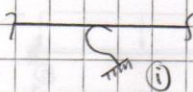
$$\sum M_i = 0$$

$$M_{ij} + M_{il} = 0$$

## • RESORTE HELICOIDAL (100 por DEB LLEGAR A UN NUDO)



## • RESORTE ESPIRAL

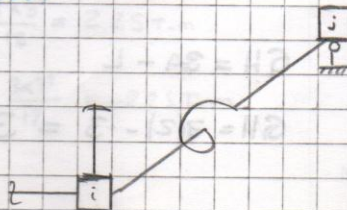
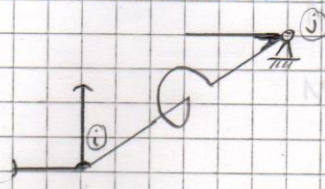


$$\theta_i = ?$$

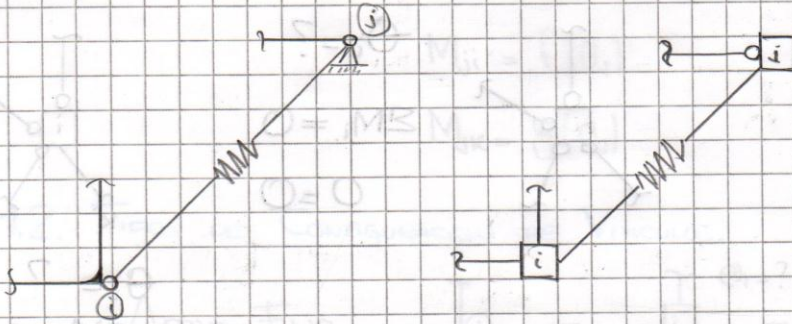
$$\sum M_i = 0$$

$$M_{ij} + M_{iK} + M^r = 0$$

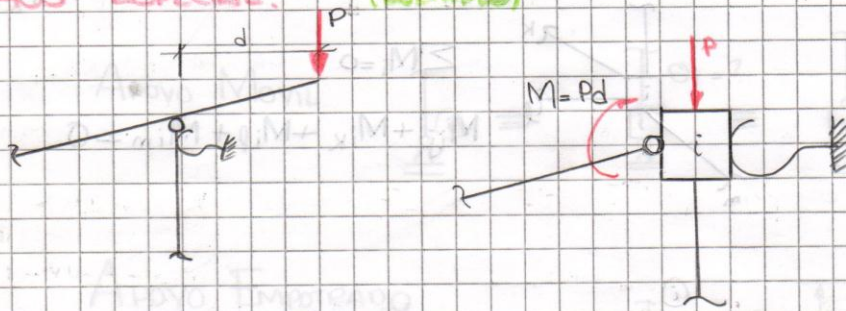
## • RESORTE ESPIRAL ENTRE DOS NUDOS



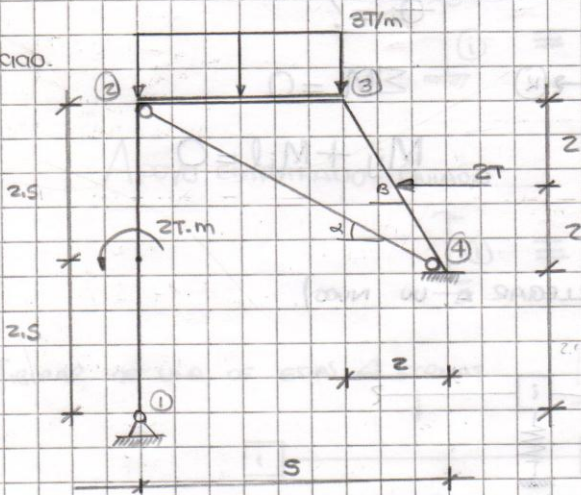
# RESORTE HELICOIDAL ENTRE DOS NUDOS



## CASO ESPECIAL (EJEMPLO)



EJERCICIO.



DATOS  
 $T_1$  30/40 (cm)  
 $T_2$  30/30 (cm)  
 $E = 21ES$  (Kg/cm<sup>2</sup>)

$$EI = \frac{21EG \times 0.30 \times 0.40^3}{12} = 3360 \text{ (T-m}^2\text{)}$$

$$EI = \frac{21EG \times 0.30 \times 0.30^3}{12} = 1417.5 \text{ (T-m}^2\text{)}$$

SOLUCION

### 1. CARACTERISTICAS GEOMETRICAS

$$\alpha = \arctg\left(\frac{4}{2.5}\right) = 58.659$$

$$l_{24} = \sqrt{4^2 + 2.5^2}$$

$$\beta = \arctg\left(\frac{4}{2}\right) = 63.435$$

$$l_{34} = \sqrt{4^2 + 2^2}$$

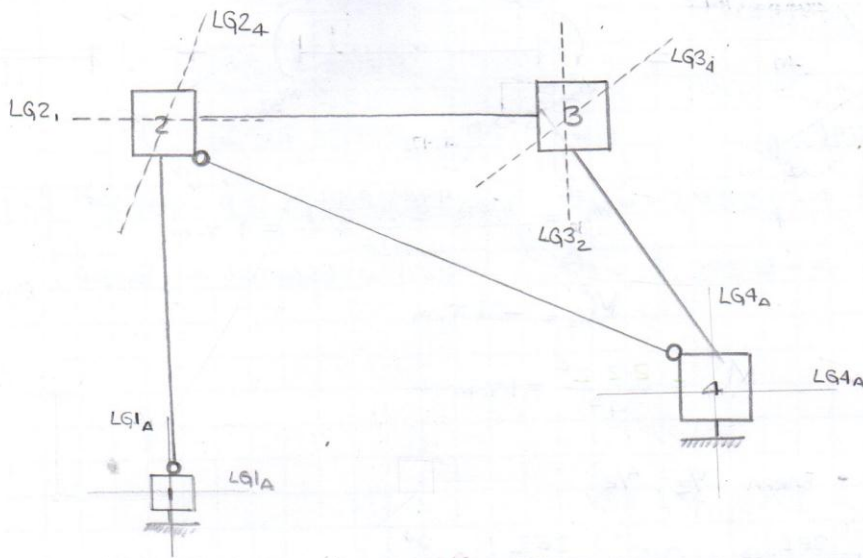
### 2. GRADOS DE HIPERESTATICIDAD

$$GH = 3A - L$$

$$GH = 3(2) - 3 = 3$$



### 3. CONFIGURACION DE VINCULOS.



3.1. GRADO DE DESPLAZABILIDAD

$$GD = ND - A = 2 - 2 = 0$$

NO EXISTE DESPLAZAMIENTOS LINEALES EN NUESTROS NUDOS

3.2. LUGARES GEOMETRICOS

3.3. IDENTIFICACION DE DATOS E INCOGNITAS.

DATOS

$$\theta_1 = 0$$

$$\theta_4 = 0$$

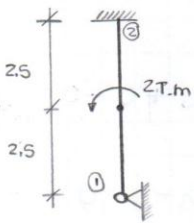
INCOGNITAS

$$\theta_2 = ? \quad \sum M_2 = 0 \quad \text{--- (1)}$$

$$\theta_3 = ? \quad \sum M_3 = 0 \quad \text{--- (2)}$$

### 4. CALCULO DE MOMENTOS FINOS.

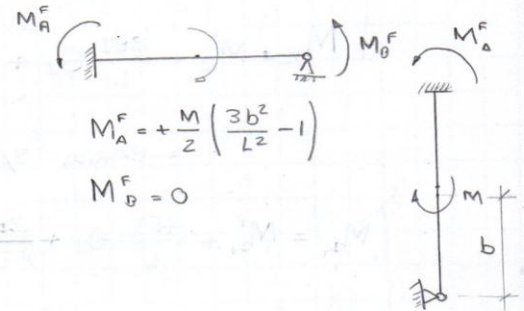
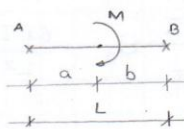
BARRA 1/2 A/E



$$M_{21}^F = + \frac{(-2)}{2} \left( \frac{3 * 2.5^2}{5^2} - 1 \right)$$

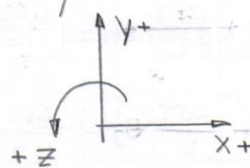
$$M_{21}^F = 0.25 \text{ T.m}$$

$$M_{12}^F = 0$$

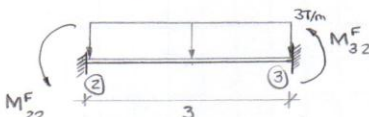


$$M_A^F = + \frac{M}{2} \left( \frac{3b^2}{L^2} - 1 \right)$$

$$M_B^F = 0$$

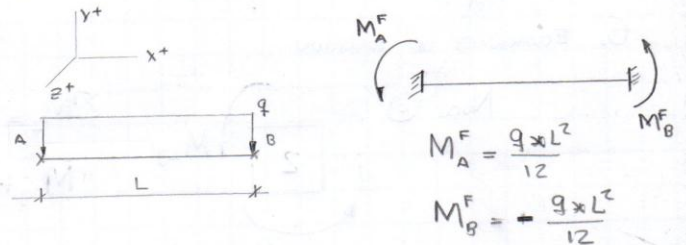


BARRA 2/3 E/E



$$M_{23}^F = \frac{3 * 3^2}{12} = 2.25 \text{ T.m}$$

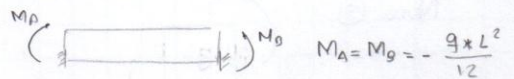
$$M_{32}^F = - \frac{3 * 3^2}{12} = -2.25 \text{ T.m}$$



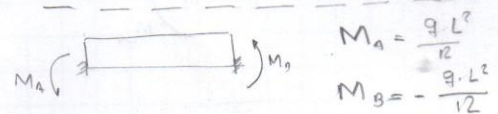
$$M_A^F = \frac{q * L^2}{12}$$

$$M_B^F = - \frac{q * L^2}{12}$$

BARRA CARGADA



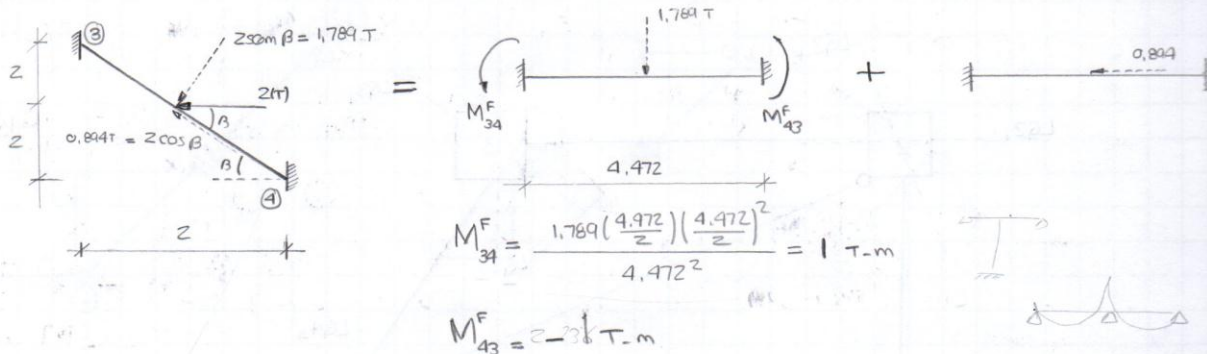
$$M_A = M_B = - \frac{q * L^2}{12}$$



$$M_A = \frac{q * L^2}{12}$$

$$M_B = - \frac{q * L^2}{12}$$

BARRA 3/4 E/E  $L_{34} = 4.472 (m)$



OTRA FORMA DE VERIFICAR.  $M_{34}^F = \frac{2 \cdot 2 \cdot 2^2}{4^2} = 1 T \cdot m$

5. MOMENTOS TOTALES • BARRA 1/2 A/E

$$\begin{cases} M_{21} = M_{21}^F + \frac{3EI}{L} \theta_2 + 0 \theta_1 - \frac{3EI}{L^2} \Delta_{21} \\ M_{12} = 0 \end{cases} \quad \begin{cases} M_{21} = 0.25 + 2016 \theta_2 + 0 + 0 \\ M_{12} = 0 \end{cases}$$

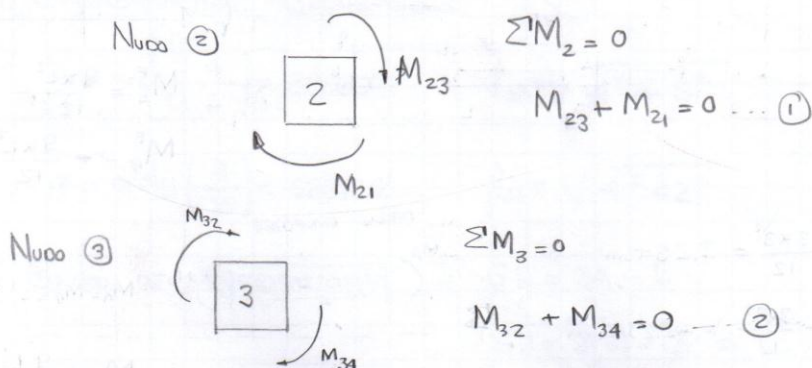
• BARRA 2/3 E/E

$$\begin{cases} M_{23} = M_{23}^F + \frac{4EI}{L} \theta_2 + \frac{2EI}{L} \theta_3 - \frac{6EI}{L^2} \Delta_{23} \\ M_{32} = M_{32}^F + \frac{2EI}{L} \theta_2 + \frac{4EI}{L} \theta_3 - \frac{6EI}{L^2} \Delta_{23} \end{cases} \quad \begin{cases} M_{23} = 2.25 + 1890 \theta_2 + 945 \theta_3 - 0 \\ M_{32} = -2.25 + 945 \theta_2 + 1890 \theta_3 - 0 \end{cases}$$

• BARRA 3/4 E/E

$$\begin{cases} M_{34} = M_{34}^F + \frac{4EI}{L} \theta_2 + \frac{2EI}{L} \theta_3 - \frac{6EI}{L^2} \Delta_{34} \\ M_{43} = M_{43}^F + \frac{2EI}{L} \theta_2 + \frac{4EI}{L} \theta_3 - \frac{6EI}{L^2} \Delta_{34} \end{cases} \quad \begin{cases} M_{34} = 1 + 1267.899 \theta_3 + 0 - 0 \\ M_{43} = -1 + 633.945 \theta_3 + 0 - 0 \end{cases}$$

6. ECUACIONES DE CONDICION



## 7. CÁLCULO DE INCOGNITAS

$$\sum M_2 = 0$$

$$2,25 + 1890\theta_2 + 945\theta_3$$

$$0,25 + 2016\theta_2$$

$$2,5 + 3906\theta_2 + 945\theta_3 = 0$$

$$\sum M_3 = 0$$

$$-2,25 + 945\theta_2 + 1890\theta_3$$

$$1$$

$$+ 1267,889\theta_3$$

$$-1,25 + 945\theta_2 + 3157,889\theta_3 = 0$$

$$3906\theta_2 + 945\theta_3 = -2,5$$

$$\theta_2 = -7,932 E - 4 \text{ (rad)}$$

$$945\theta_2 + 3157,889\theta_3 = 1,25$$

$$\theta_3 = 6,332 E - 4 \text{ (rad)}$$

