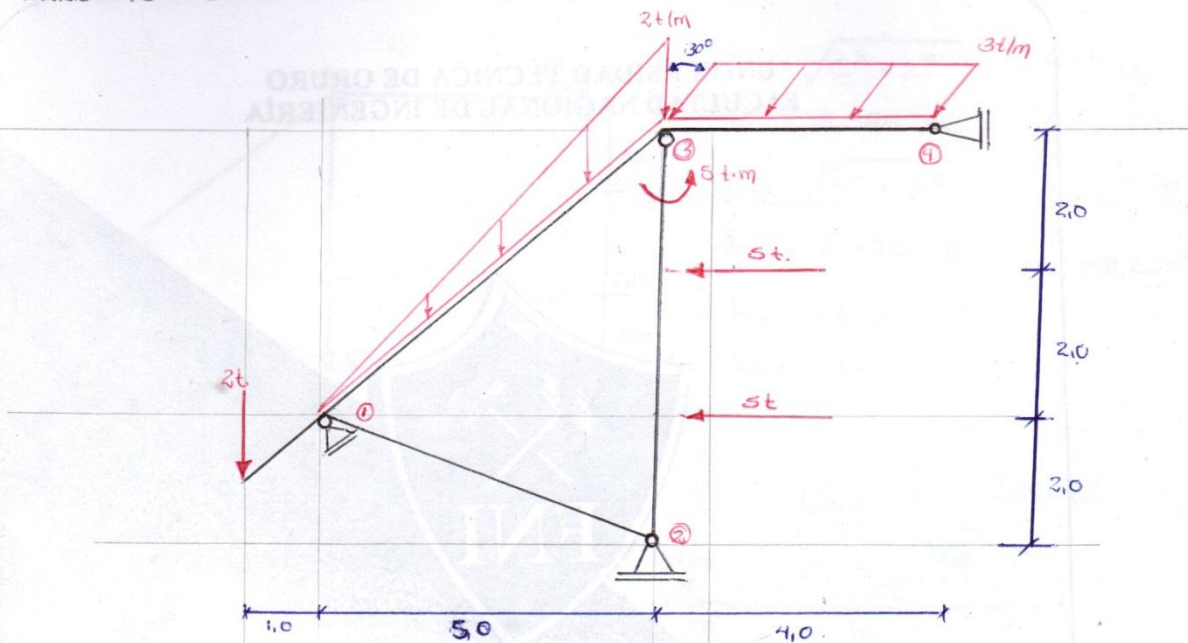


ANÁLISIS ESTRUCTURAL

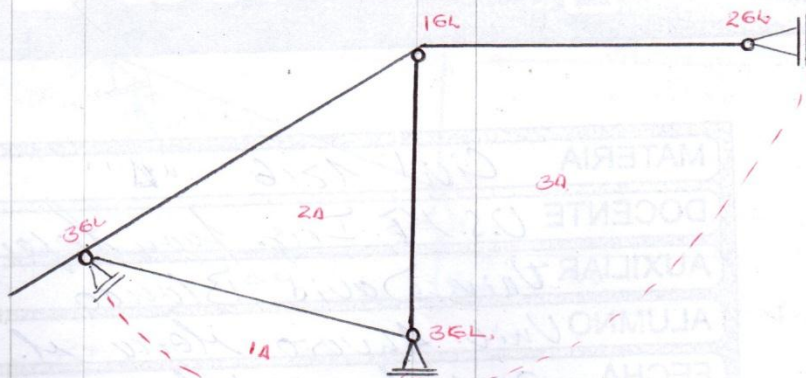
CIV-2206

PROB. HALLAR LOS DIAGRAMAS DE  $M_z$ ,  $Q_z$ ,  $N_z$  (CON DESMEMBRAMIENTO)



SOLUCIÓN

I: CALCULO DEL GRADO HIPERESTATICO



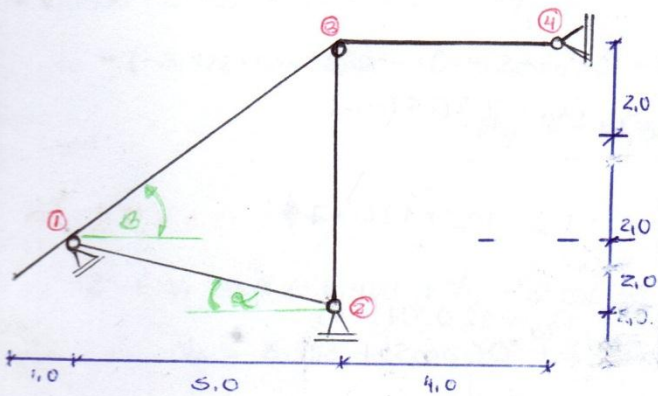
$$GH = 3A - 6L$$

$$GH = 3 \cdot 3 - 9$$

$$GH = 0$$

LA ESTRUCTURA ES ISOSTÁTICO

II CARACTERÍSTICAS GEOMÉTRICAS



$$L_{12} = \sqrt{5^2 + 2^2} \quad \alpha = \tan^{-1} \frac{2}{5}$$

$$L_{12} = 5,385 \text{ m} \quad \alpha = 21,801^\circ$$

$$L_{13} = \sqrt{4^2 + 5^2} \quad \beta = \tan^{-1} \frac{4}{5}$$

$$L_{13} = 6,403 \text{ m} \quad \beta = 38,660^\circ$$

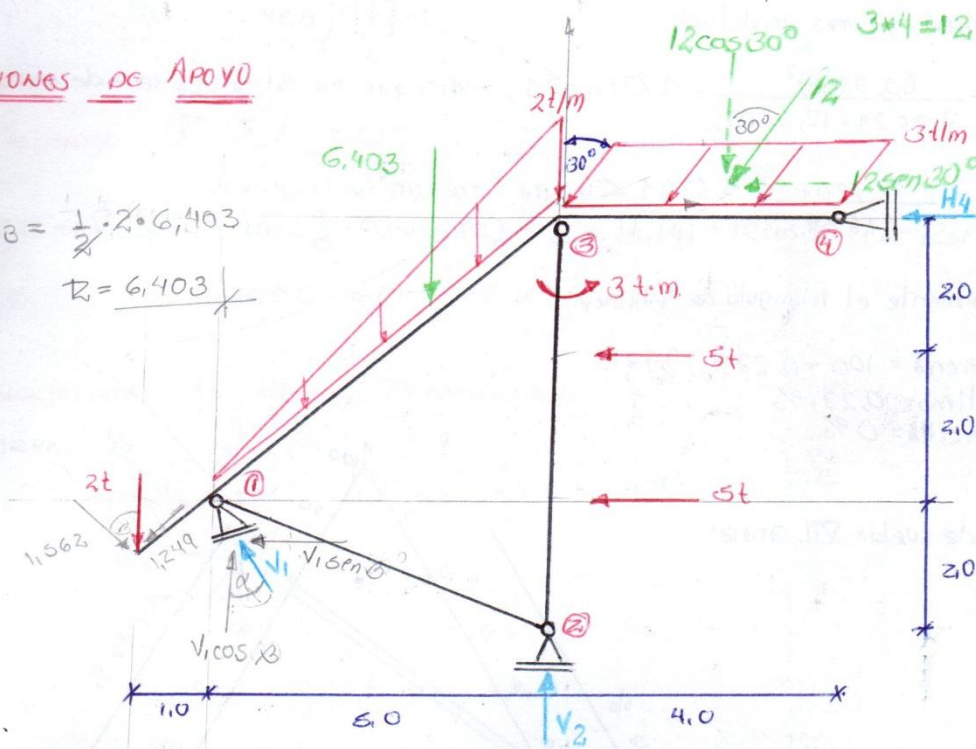
$$L_{23} = 6 \text{ m.}$$

$$L_{34} = 4 \text{ m}$$

III REACCIONES DE APOYO

$$R = \frac{1}{2} q \cdot L_{13} = \frac{1}{2} \cdot 2 \cdot 6,403$$

$$R = 6,403$$



a)  $\sum M_3 = 0 \quad (+)$

$$-2 \cdot 6 + V_1 \cdot L_{13} - 6,403 \left( \frac{1}{3} \cdot 3 \right) - 3 + 5 \cdot 2 + 5 \cdot 4 + 12 \cos 30^\circ (2) = 0$$

$$V_1 = -\frac{25,113}{6,403} \quad \therefore V_1 = -3,922 \text{ t}$$

ANÁLISIS ESTRUCTURAL  
CIV-2205

b)  $\Sigma FH = 0 \rightarrow$

$$-V_1 \sin 30^\circ - 5 - 5 - 12 \sin 30^\circ - H_4 = 0$$

$$-(-3,922 \cdot \sin 38,66^\circ) - 10 - 12 \sin 30^\circ - H_4 = 0$$

$$H_4 = -13,55 \text{ [t]}$$

c)  $\Sigma FV = 0 \uparrow$

$$-2 + V_1 \cos \alpha - 6,403 + V_2 - 12 \cos 30^\circ$$

$$V_2 = 8,403 + 12 \cos 30^\circ + 3,922 \cdot \cos 38,66^\circ$$

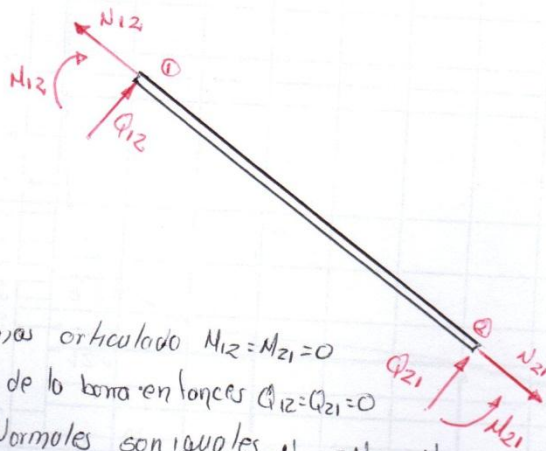
$$V_2 = 21,858 \text{ [t]}$$

d) CONTROL.  $\Sigma M_1 = 0$

$$-2(1) - V_2(6) + 6,403\left(\frac{2}{3} \cdot 6\right) - 5(2) - 3 - H_4(4) + 12 \cos 30^\circ(7) - 12 \sin 30^\circ(4) = 0$$

$$-0,0005 = 0 \quad \text{OK!}$$

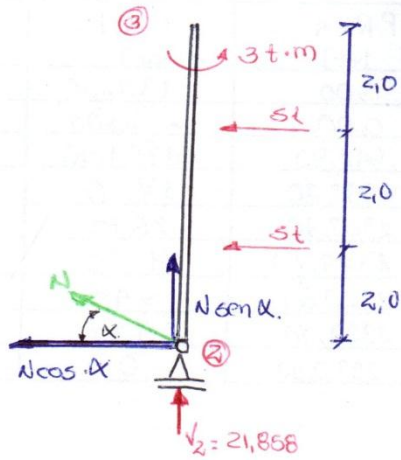
e) ANALIZANDO LA BARRA BINEUTICULAR  
BARRA  $\frac{1}{2}$



- \* Se tiene extremos articulados  $N_{12} = N_{21} = 0$
- \* No existen cargas de la barra en lonces  $Q_{12} = Q_{21} = 0$
- \* Se tiene q' las Normales son iguales  $N_{12} = N_{21} = N$

ANÁLISIS ESTRUCTURAL  
CIV-2205

→ CALCULAR  $N$ .



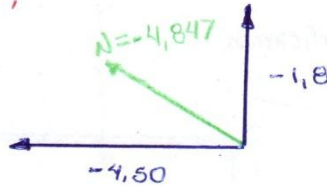
$$\sum N_3 = 0 \quad (+)$$

$$N \cdot \cos \alpha \cdot (6) + 5(4) + 5(2) - 3 = 0$$

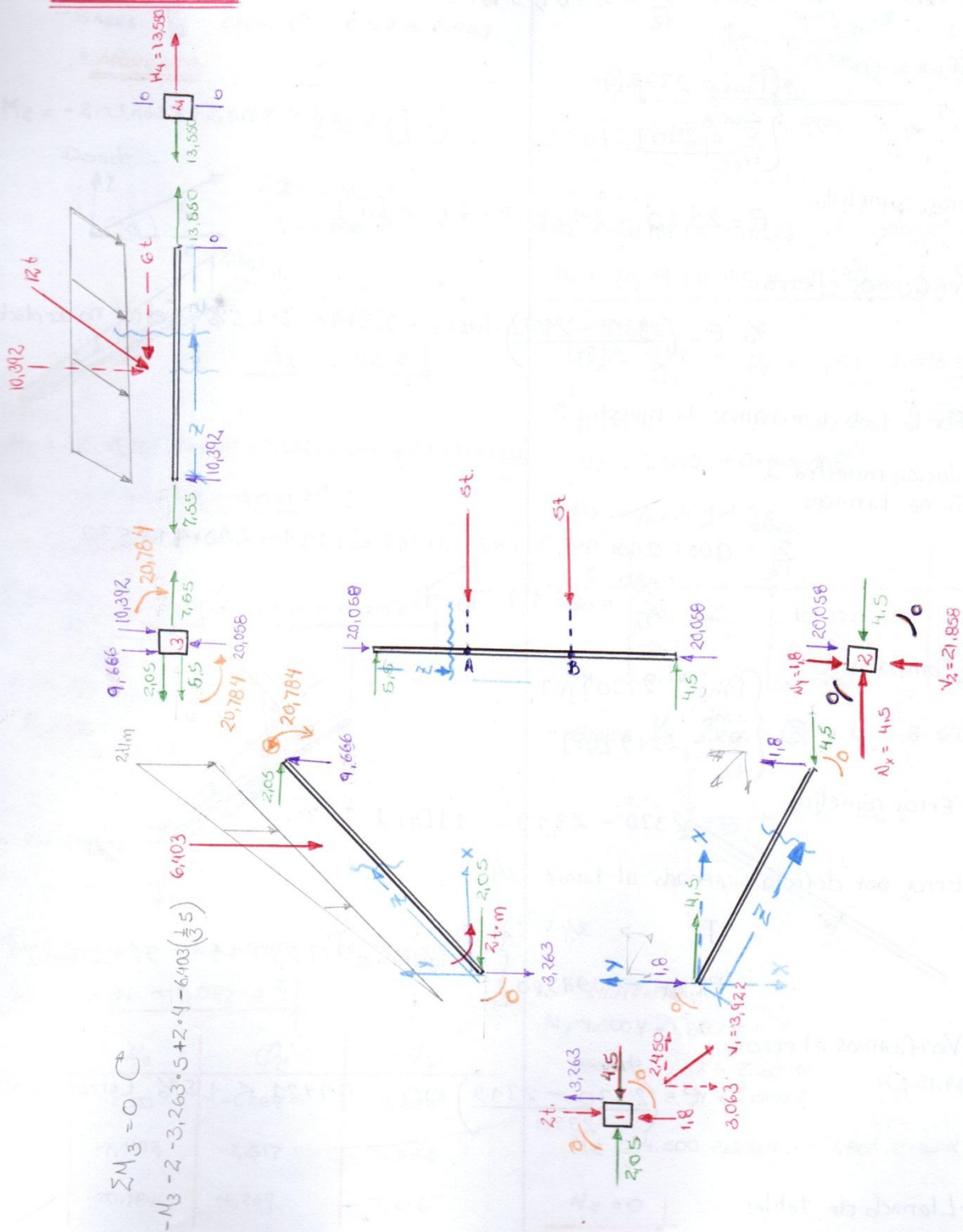
$$N \cdot \cos \alpha \cdot (6) + 27 = 0$$

$$N = \frac{27}{\cos 21,801(6)}$$

$$N = -4,847$$



DESMEMBRO



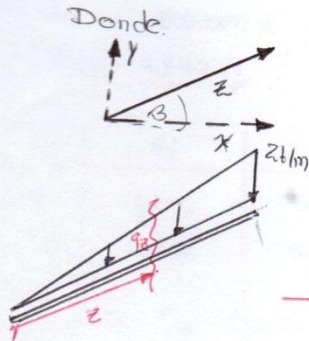
V FUNCIÓN DE FUERTAS INTERNAS

- BARRA 1/3 ORIG. ①  $0 \leq z \leq 6,403$

• MOMENTO.

$$M_z = -2 - 3,263x + 2,05y - \frac{1}{2} q_z \cdot z \left( \frac{1}{3} \cdot z \right)$$

Donde.



$$x = z \cos \beta$$

$$y = z \sin \beta$$

$$\frac{q_z}{z} = \frac{2}{6,403}$$

$$q_z = 0,312 z$$

$$M_z = -2 - 3,263(z \cos \beta) + 2,05(z \sin \beta) - \frac{1}{2} \cdot 0,312 z \cdot z \left( \frac{z \cos \beta}{3} \right)$$

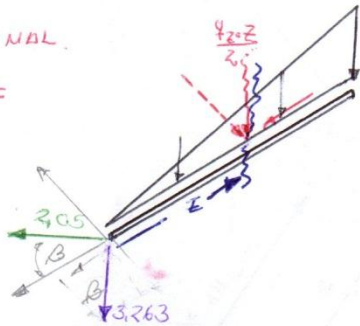
$$M_z = -2 - 1,267 z - 0,041 z^3$$

• CORTANTE

$$Q_z = \frac{dM_z}{dz} ; Q_z = -1,267 z - 0,123 z^2$$

• NORMAL

$$N_z =$$

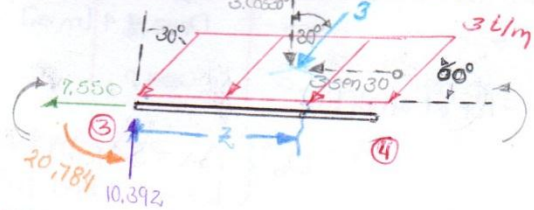


$$N_z = 2,05 \cos \beta + 3,263 \sin \beta + \frac{q_z \cdot z}{2} \cdot \sin \beta$$

$$N_z = 3,639 + 0,097 z^2$$

z	M <sub>z</sub>	Q <sub>z</sub>	N <sub>z</sub>
0	-2,00	-1,267	3,639
3,2015	-7,402	-2,517	4,832
6,403	-20,784	-6,269	7,616

- BARRA 3/4 ORIG EN. ③  $0 \leq z \leq 4$



• MOMENTO.

$$+M_z = -20,784 + 10,392(z) - 3z \cos 30^\circ \left( \frac{z}{2} \right)$$

$$M_z = -20,784 + 10,392 z - 1,299 z^2$$

• CORTANTE.

$$Q_z = \frac{dM_z}{dz} ; Q_z = 10,392 - 2,598 z$$

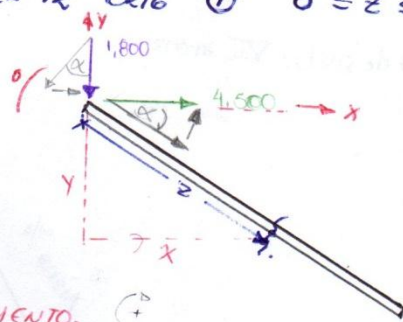
• NORMAL.

$$N_z = 7,550 + 3 \cdot z \sin 30^\circ$$

$$N_z = 7,550 + 1,5 z$$

z	M <sub>z</sub>	Q <sub>z</sub>	N <sub>z</sub>
0	-20,784	10,392	7,550
2	-5,196	5,196	10,550
4	0,000	0,000	13,650

- BARRA 1/2 ORIG ①  $0 \leq z \leq 5,385$



• MOMENTO.

$$M_z = 4,500 y - 1,800 x$$

Donde.

$$x = z \cos \alpha$$

$$y = z \sin \alpha$$

$$\alpha = 21,801^\circ$$

$$M_z = 4,500 z \sin \alpha - 1,800 z \cos \alpha$$

$$M_z = 0$$

ANÁLISIS ESTRUCTURAL  
CIV. - 2205

• **CONSTANTE:**

$$Q_z = \frac{dM_z}{dz} ; \phi_z = 0$$

• **NORMAL:**

$$M_z = -1.800 \operatorname{sen} \alpha - 4.500 \operatorname{cos} \alpha$$

$$N_z = -4.847$$

z	M <sub>z</sub>	Q <sub>z</sub>	N <sub>z</sub>
0	0,000	0,000	-4,847
5,385	0,000	0,000	-4,847

- BARRA 2/3 ORIG. (3)

\* TRAMO 3/A. 0 ≤ z ≤ 2

• **MOMENTO**

$$M_z = -3 + 5.500z$$

• **CONSTANTE.**

$$Q_z = 5.500$$

• **NORMAL.**

$$N_z = -20.058$$

z	M <sub>z</sub>	Q <sub>z</sub>	N <sub>z</sub>
0	-3,000	5,500	-20,058
2	8,000	5,500	-20,058

\* TRAMO A/B 2 ≤ z ≤ 4

• **MOMENTO**

$$M_z = -3 + 5.500z - 5(z-2)$$

$$M_z = 7 + 0,5z$$

• **CONSTANTE.**

$$Q_z = \frac{dM_z}{dz} ; \phi_z = 0,5$$

• **NORMAL.**

$$N_z = -20.058$$

z	M <sub>z</sub>	Q <sub>z</sub>	N <sub>z</sub>
3	8,000	0,500	-20,058
4	9,000	0,500	-20,058

\* TRAMO B/2. 4 ≤ z ≤ 6

• **MOMENTO**

$$M_z = -3 + 5.5z - 5(z-2) - 5(z-4)$$

$$M_z = 27 - 4,5z$$

• **CONSTANTE**

$$Q_z = \frac{dM_z}{dz} ; \phi_z = -4,5$$

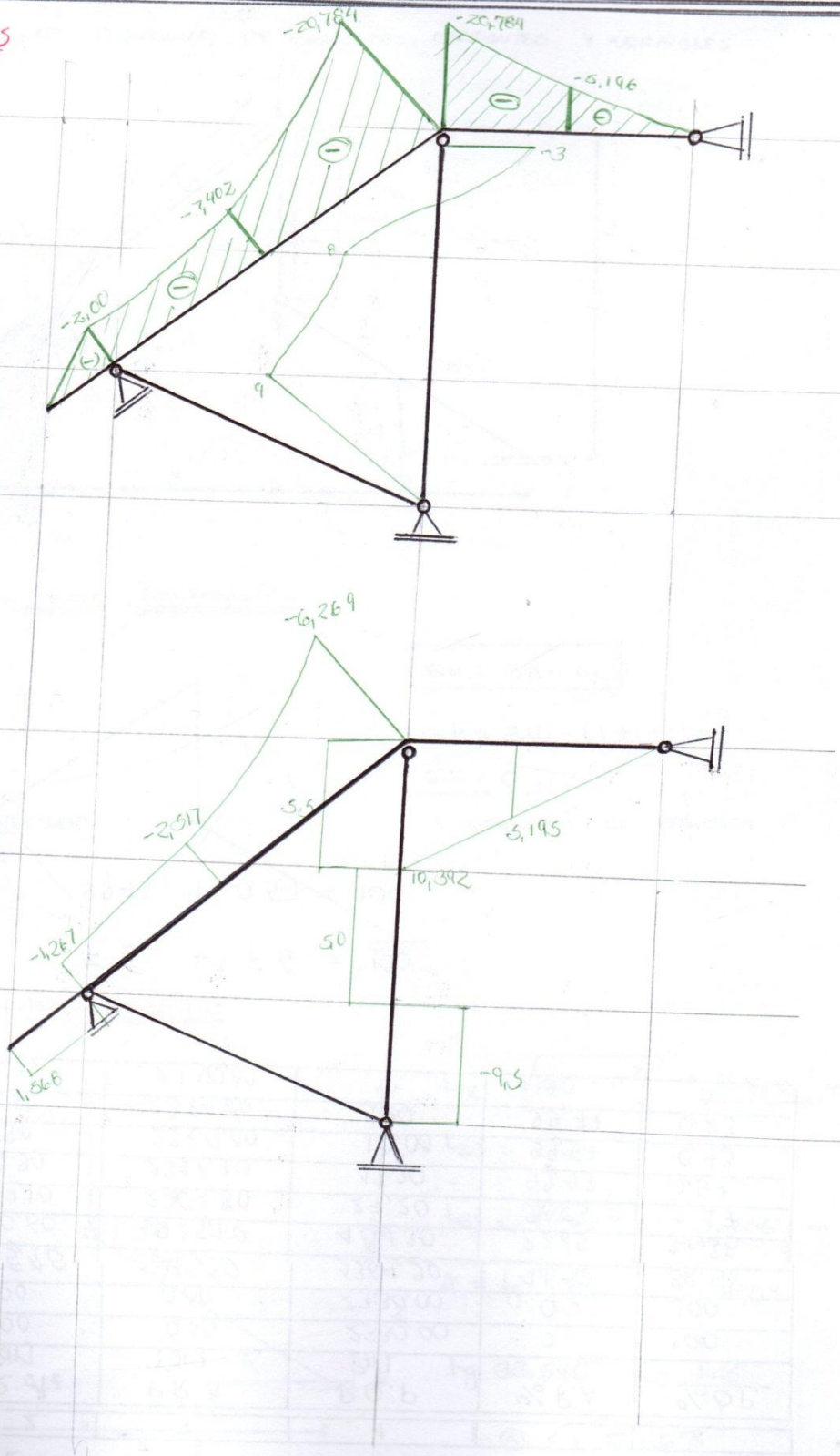
• **NORMAL.**

$$N_z = -20.058$$

z	M <sub>z</sub>	Q <sub>z</sub>	N <sub>z</sub>
4	9,000	-4,500	-20,058
6	0,000	-4,500	-20,058

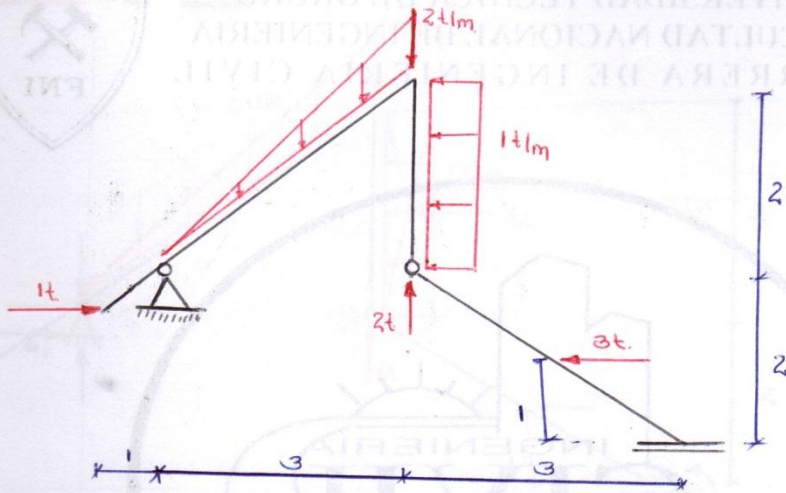
ANÁLISIS ESTRUCTURAL  
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VI: DIAGRAMAS  
• MOMENTOS



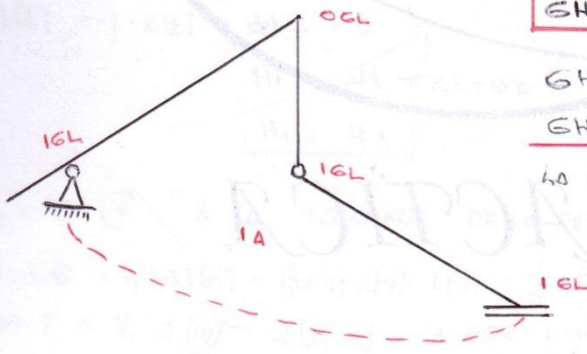


Prob. 1: Hallar los diagramas de momentos, cortantes y normales



SOLUCIÓN

I: CÁLCULO DEL GRADO HIPERESTÁTICO



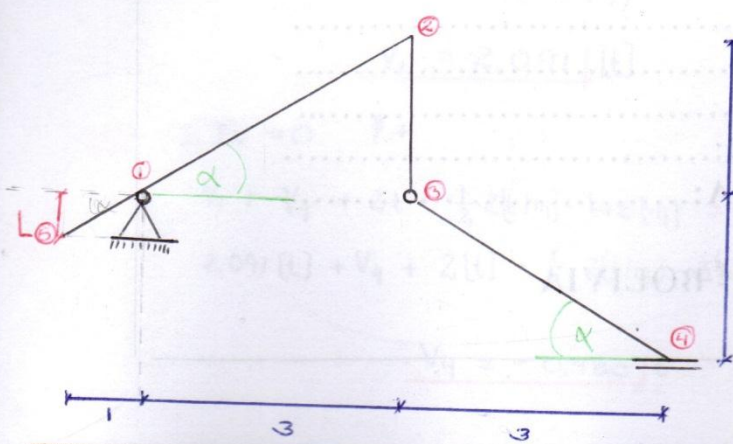
$$GH = 3A - 6L$$

$$GH = 3(1) - (1+1+1)$$

$$GH = 0$$

LA ESTRUCTURA ES ESTÁTICA

II: CARACTERÍSTICAS GEOMÉTRICAS



$$L_{12} = \sqrt{2^2 + 3^2} = 3,606 \text{ m}$$

$$L_{23} = 2 \text{ m}$$

$$L_{34} = \sqrt{3^2 + 2^2} = 3,606 \text{ m}$$

$$\alpha = \text{tg}^{-1}\left(\frac{2}{3}\right) = 33,690^\circ$$

$$\text{tg } 33,690^\circ = \frac{L_5}{1}$$

$$L_5 = 0,667$$



ANÁLISIS ESTRUCTURAL  
AN CIV- 2205.

d)  $\sum M_3 = 0$   $\odot$  "A LA DERECHA DE LA ART."

$$-V_4 \cdot 3[m] - M_4 + 3[t] \cdot 1[m] = 0$$

$$-M_4 = -0,485 \cdot 3 - 3$$

$$\underline{M_4 = 4,455} [t \cdot m]$$

e) CONTROL

$$\sum M_i = 0 \quad \odot$$

$$-1[t] \cdot 0,667[m] + \frac{1}{2} \cdot 2 [t/m] \cdot 3,606 m \cdot \frac{2}{3} \cdot 3 m - 2[t] \cdot 3[m] - 1[t/m] \cdot 2[m] \cdot 1[m]$$

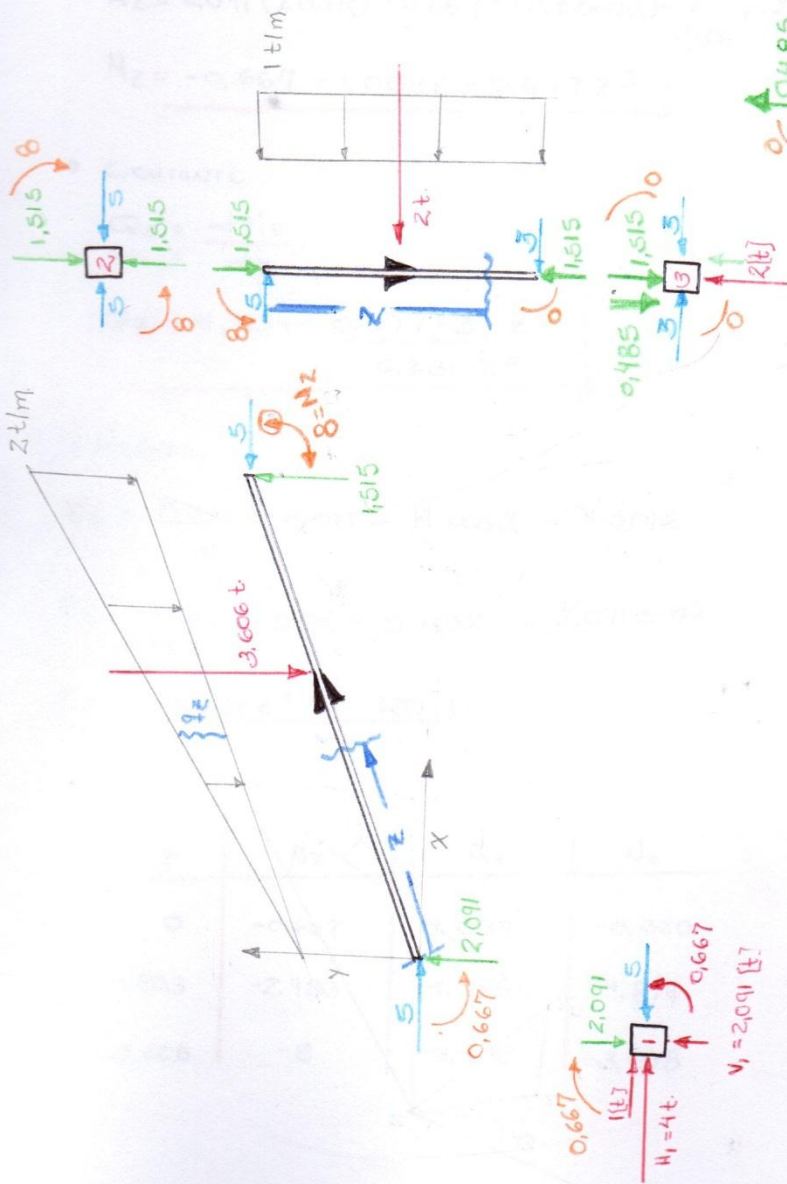
$$+ 3[t] \cdot 1[m] - V_4 \cdot 6[m] - M_4 = 0$$

$$-0,667[t \cdot m] + 7,212[t \cdot m] - 6[t \cdot m] - 2[t \cdot m] + 3[t \cdot m] + 2,91 - 4,455 = 0$$

$$\underline{0=0} \quad \text{OK!}$$

ANALISIS ESTRUCTURAL  
CIV - 2205

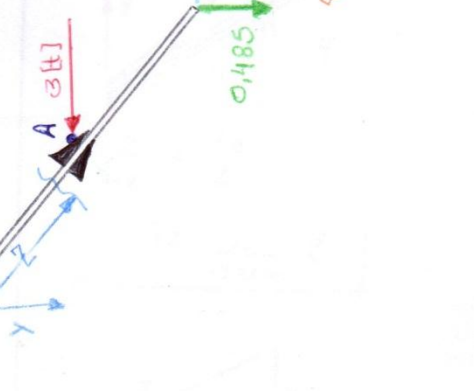
IV DESMEMBRAMIENTO



$$\sum M_4 = 0 \oplus$$

$$3 \cdot 2 + 0,485 \cdot 3 - 3 \cdot 1 \quad M_4 = 0$$

$$M_4 = 4,485$$



$$\sum M_2 = 0 \oplus$$

$$2,091 \cdot 3 - 0,667 - 5 \cdot 2 - \frac{1}{2} \cdot 2 \cdot 3,606 \cdot \frac{1}{3} \cdot 3 - M_2 = 0$$

$$M_2 = -8$$

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ANÁLISIS ESTRUCTURAL  
C.W. 2205

\* BARRA 2/3 ORIGEN (2)  $0 \leq z \leq 2$

o MOMENTO.  $\curvearrowright$

$$M_z = -8 + 5z - \frac{1}{2}z \cdot \frac{z^2}{2}$$

$$M_z = -8 + 5z - \frac{z^3}{2}$$

o CORTANTE:

$$Q_z = 5 - z$$

$$Q_z = \frac{dM_z}{dz}$$

o NORMALES.

$$N_z = -1,515$$

z	Mz	Qz	Nz
0	-8	5	-1,515
1	-3,5	4	-1,515
2	0	3	1,515

\* BARRA 3/4 ORIGEN (3)

→ TRAMO 3/A  $0 \leq z \leq 1,803$

o MOMENTO.  $\curvearrowright$

$$M_z = 3y + 0,485x$$

$$M_z = 3(z \cdot \sin \alpha) + 0,485(z \cdot \cos \alpha)$$

$$M_z = 2,068z$$

o CORTANTE

$$Q_z = \frac{dM_z}{dz} ; Q_z = 2,068$$

o NORMAL.

$$N_z = 0,485 \sin \alpha - 3 \cos \alpha$$

$$N_z = -2,227$$

z	Mz	Qz	Nz
0	0	2,068	-2,227
1,803	3,729	2,068	-2,227

→ TRAMO A/4  $1,803 \leq z \leq 3,606$

o MOMENTO.

$$M_z = 3y + 0,485x - 3(y-1)$$

$$M_z = 0,485z \cos \alpha + 3$$

$$M_z = 0,404z + 3$$

o CORTANTE.

$$Q_z = \frac{dM_z}{dz} ; Q_z = 0,404$$

NORMAL.

$$N_z = 0,485 \cos \alpha$$

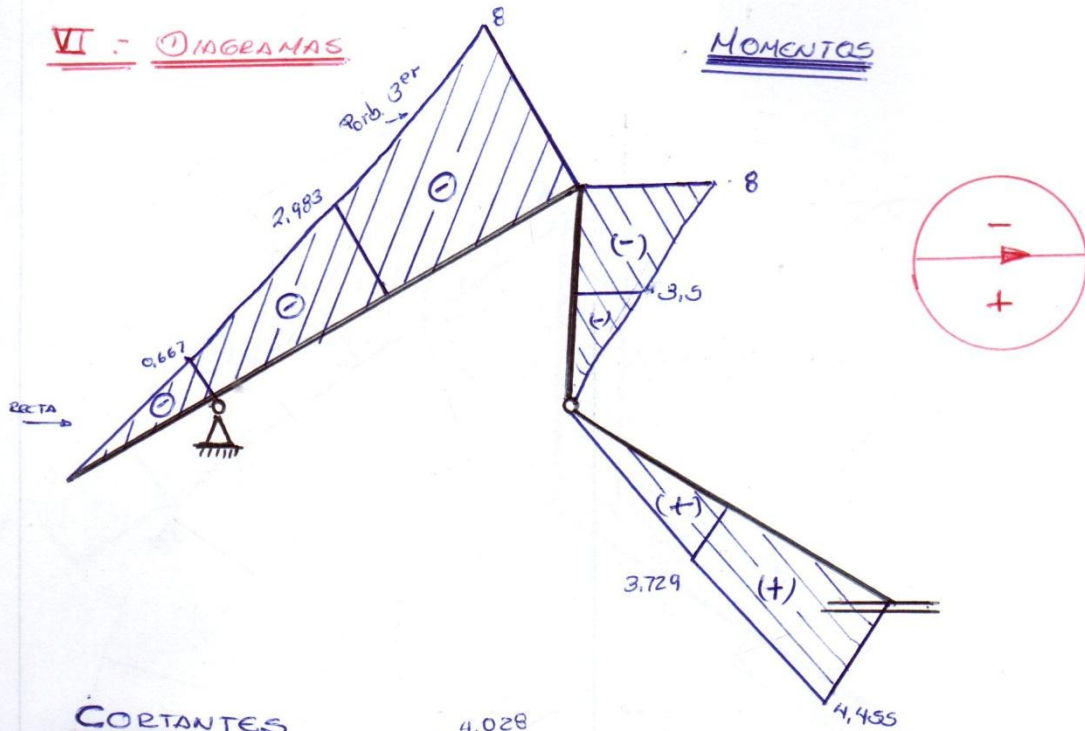
$$N_z = 0,269$$

z	Mz	Qz	Nz
1,803	3,729	0,404	0,269
3,606	4,456	0,404	0,269

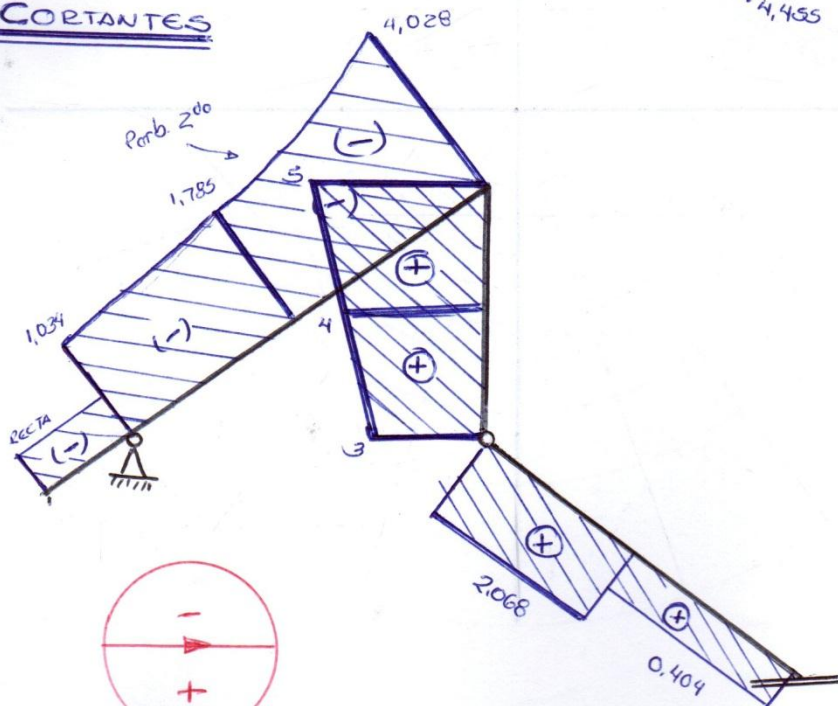
ANÁLISIS ESTRUCTURAL  
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VI - DIAGRAMAS

MOMENTOS

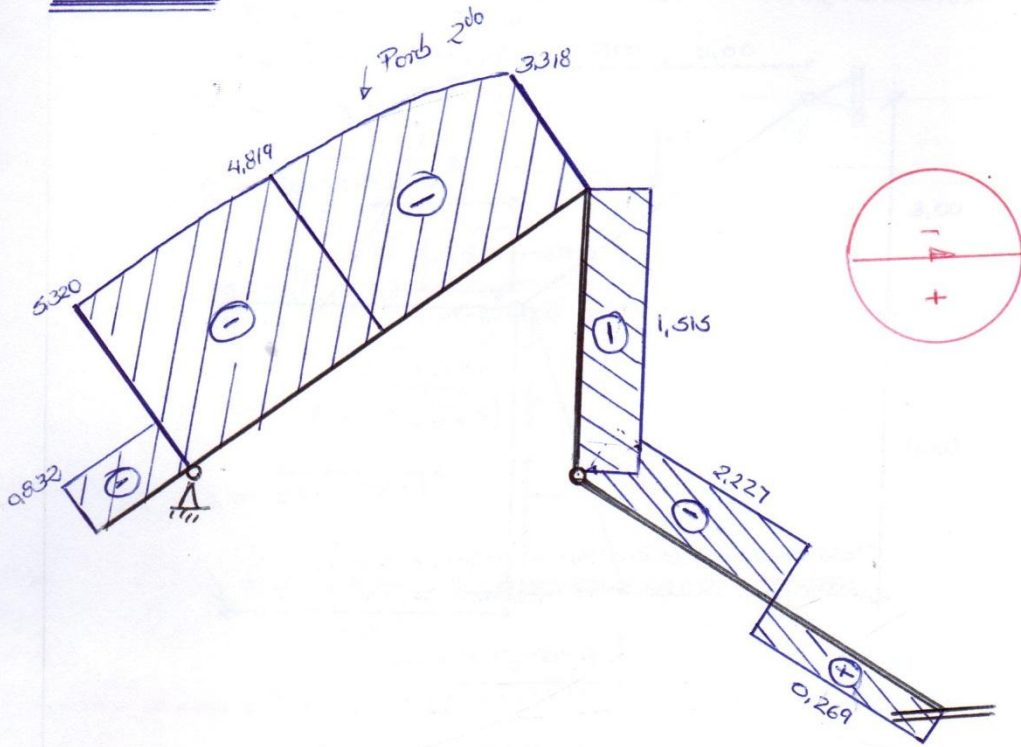


CORTANTES



ANALISIS ESTRUCTURAL  
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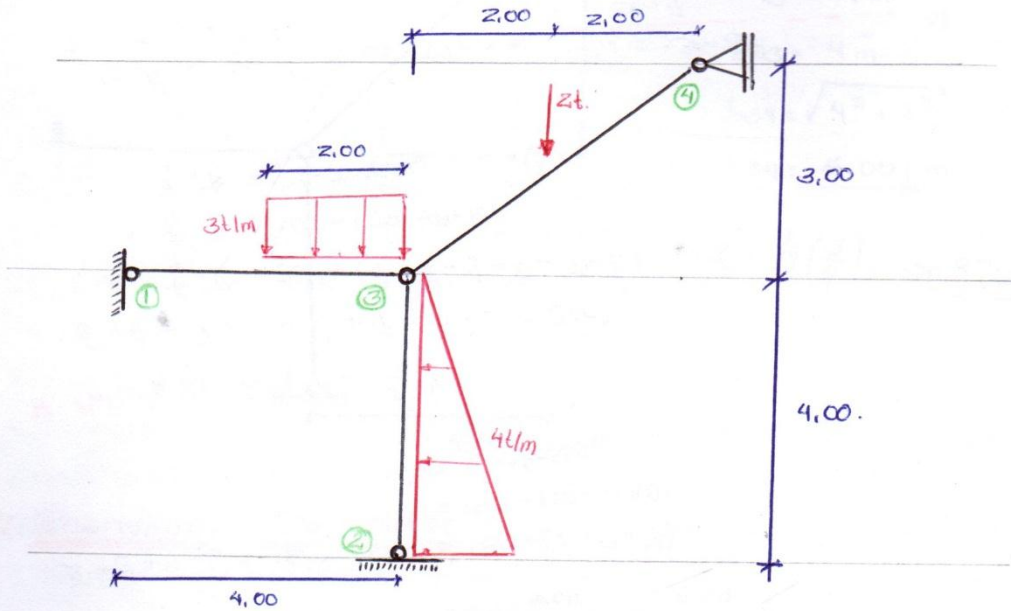
NORMALES





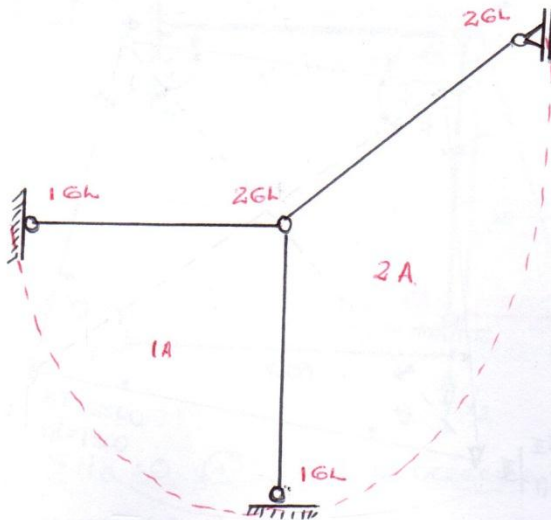
ANÁLISIS ESTRUCTURAL  
CIV-2205

PROB. HALLAR DIAGRAMAS DE  $M_z$ ,  $Q_z$ ,  $N_z$ , (DESMEMBRAMIENTO)



SOLUCIÓN

I: CÁLCULO DEL GRADO HIPERESTÁTICO



$$GH = 3A - GL$$

$$GH = 3 \cdot 2 - 6$$

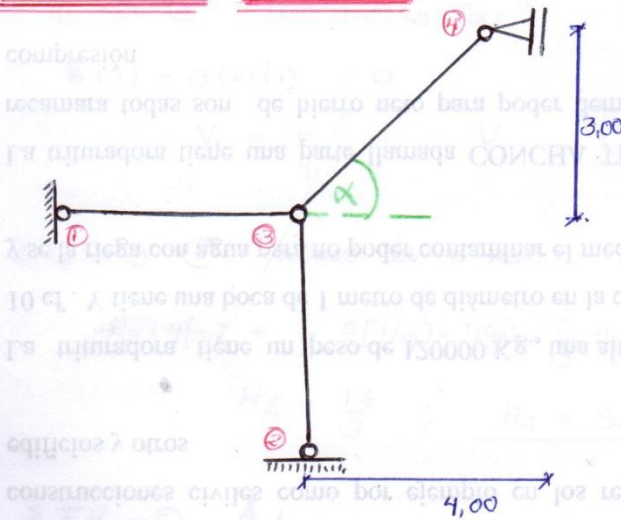
$$\underline{GH = 0}$$

La ESTRUCTURA ISOSTÁTICA

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ANÁLISIS ESTRUCTURAL  
CIV - 2205

II - CARACTERÍSTICAS GEOMÉTRICAS



$$L_{13} = 4 \text{ m}$$

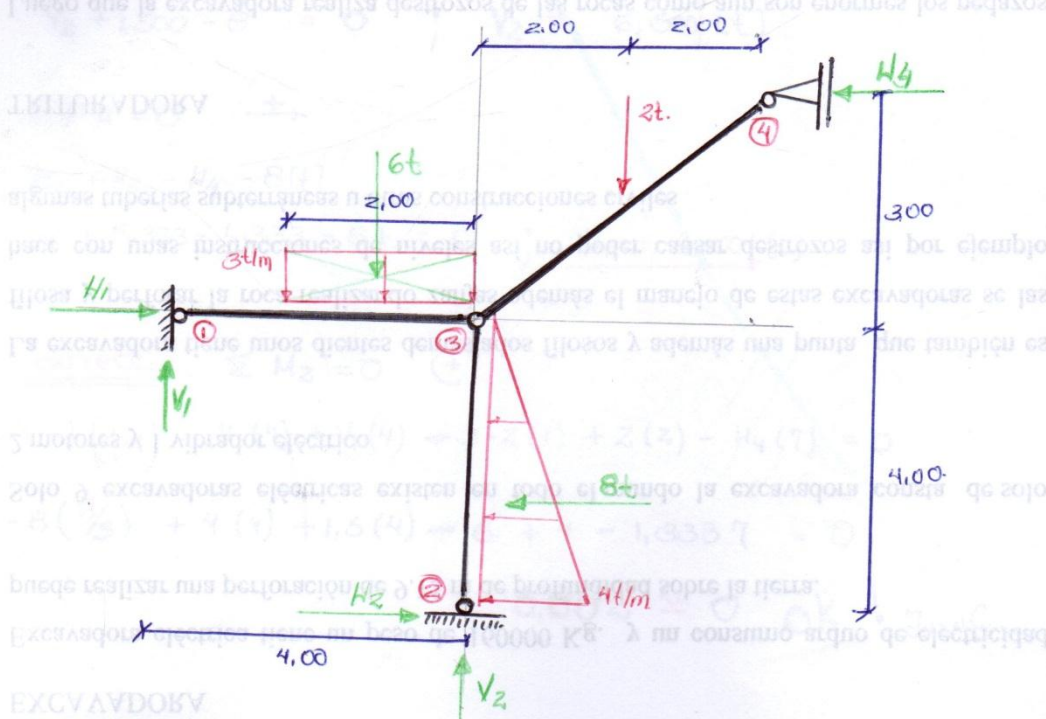
$$L_{23} = 4 \text{ m}$$

$$L_{34} = \sqrt{4^2 + 3^2}$$

$$L_{34} = 5.00 \text{ m}$$

$$\alpha = \tan^{-1}\left(\frac{3}{4}\right) = 36.870^\circ$$

III REACCIONES DE APOYO



a)  $\sum M_B = 0 \quad (\oplus)$  "A LA DERECHA DE LA ART."

$$2[t] \cdot 2\text{m} - H_4 \cdot 3[m] = 0$$

$$H_4 = \frac{4}{3} \quad ; \quad H_4 = 1.333 [t]$$

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## ANÁLISIS ESTRUCTURAL

CIV-2205

b)  $\Sigma M_3 = 0 \quad \textcircled{+}$  "IZQ de la ART"

$$V_1(4) - 3(2)(1) = 0$$

$$V_1 = \frac{6}{4} \quad ; \quad V_1 = \underline{1,500} \text{ [t]}$$

c)  $\Sigma M_3 = 0 \quad \textcircled{+}$  "ABADO de la ART."

$$-H_2 \cdot 4[m] + \frac{1}{2} 4[m] \cdot 4[m] \cdot \frac{2}{3} \cdot 4[m] = 0$$

$$H_2 = \frac{16}{3} \quad ; \quad H_2 = \underline{5,333} \text{ [t]}$$

d)  $\Sigma FV = 0 \quad \uparrow +$

$$V_1 + V_2 - 6[t] - 2[t] = 0$$

$$V_2 + 1,500 - 8 = 0 \quad ; \quad V_2 = \underline{6,500} \text{ [t]}$$

e)  $\Sigma FH = 0 \quad \rightarrow +$

$$H_1 + H_2 - H_4 - 8[t]$$

$$H_1 + 5,333 - 4,333 - 8 = 0 \quad ; \quad H_1 = \underline{4,000} \text{ [t]}$$

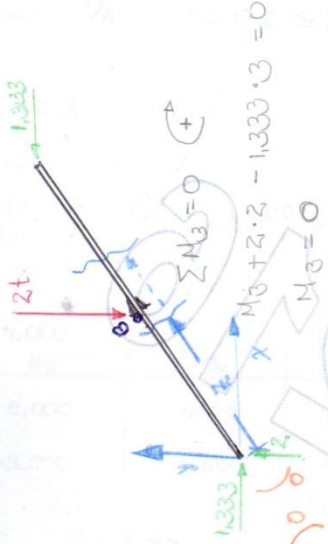
f) CONTROL  $\Sigma M_2 = 0 \quad \textcircled{+}$

$$-\frac{1}{2}(4 \cdot 4) \cdot \left(\frac{1}{3} \cdot 4\right) + H_1(4) + V_1(4) + 3 \cdot 2(1) + 2(2) - H_4(7) = 0$$

$$-8\left(\frac{4}{3}\right) + 4(4) + 1,5(4) + 6 + 4 - 1,333(7) = 0$$

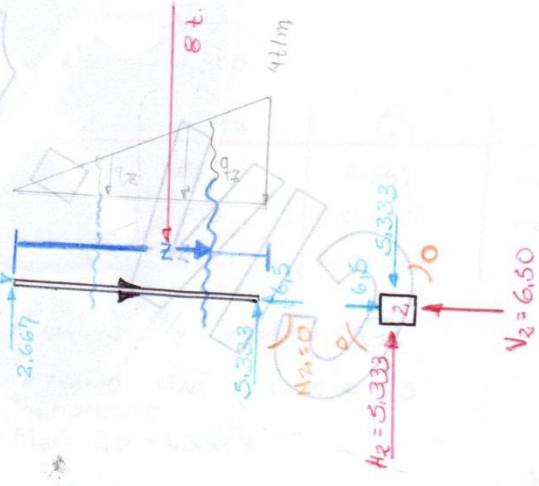
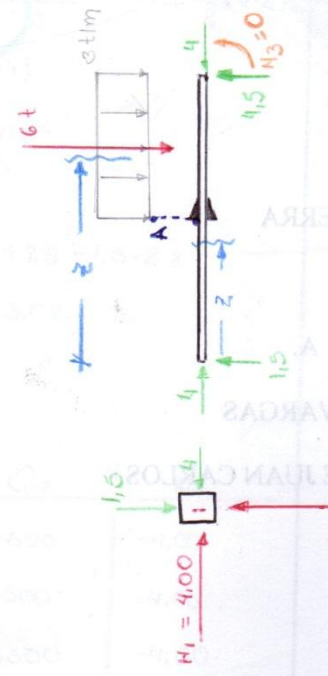
$$0,002 \approx 0 \quad \text{OK} \quad \text{! JUAN.C.}$$

ANÁLISIS ESTRUCTURAL  
 CW - 2205.



$\sum M_2 = 0$   
 $M_2 + 2,667 \cdot 4 - \frac{1}{2} \cdot 4 \cdot 4 \cdot \frac{1}{3} = 0$   
 $M_2 = 0$

$\sum M_3 = 0$   
 $1,5(4) - 3(z) - M_3 = 0$   
 $M_3 = 0$



V FUNCIONES DE FUERTAS INTERNAS

- BARRA 1/3 ORIGEN ①

\* TRAMO 1/A  $0 \leq z \leq 2$

o MOMENTO.

$M_z = 1,500 z$

o CORTANTE.

$Q_z = \frac{dM_z}{dz} ; Q_z = 1,500$

o NORMAL

$N_z = -4,000$

z	Mz	Qz	Nz
0	0,000	1,500	-4,000
2	3,000	1,500	-4,000

\* TRAMO 4/3  $2 \leq z \leq 4$

o MOMENTO.

$M_z = 1,500z - 3 \cdot (z-2) \cdot \frac{(z-2)}{2} = 1,5z - \frac{3}{2}(z-2)^2$

$M_z = 1,5z - \frac{3}{2}(z^2 - 4z + 4)$

$M_z = -6,0 + 7,5z - 1,5z^2$

o CORTANTE.

$Q_z = \frac{dM_z}{dz} ; Q_z = -6,0 + 7,5 - 1,5 \cdot 2z$

$Q_z = 7,5 - 3,0z$

o NORMAL.

$N_z = -4,00$

z	Mz	Qz	Nz
2	3,00	1,500	-4,00
3	3,00	-1,500	-4,00
4	0,00	-4,500	-4,00

- BARRA 2/3 ORIG. ③  $0 \leq z \leq 4$

o MOMENTO.

$M_z = 2,667z - \frac{1}{2} q_2 \cdot z \cdot \left(\frac{1}{3} \cdot z\right)$

$M_z = 2,667z - 0,167z^3$

o CORTANTE.  $Q_z = \frac{dM_z}{dz}$

$Q_z = 2,667 - 0,5z^2$

o NORMAL

$N_z = -6,500$

z	Mz	Qz	Nz
0	0,00	2,667	-6,500
2	4,00	0,667	-6,500
4	0,00	-6,333	-6,500

- BARRA 3/4 ORIGEN ③

\* TRAMO 3/3  $0 \leq z \leq 2,5$

o MOMENTO.

$M_z = 2x - 1,333y$

Donde

$x = z \cos \alpha$

$y = z \sin \alpha$

$\alpha = 36,870^\circ$

$M_z = 0,802z$

o CORTANTE

$Q_z = \frac{dM_z}{dz} ; Q_z = 0,802$

o NORMAL

$N_z = -2,5 \sin \alpha - 1,333 \cos \alpha$

$N_z = -2,266$

z	Mz	Qz	Nz
0	0	0,802	-2,266
2,5	2,00	0,802	-2,266

# TRAMO  $\theta/4$   $2,5 \leq z \leq 5$

**MOMENTO**

$$M_z = 2x - 1,33y - z(x-2)$$

Donde

$$x = z \cos \alpha$$

$$y = z \sin \alpha$$

$$\alpha = 36,870^\circ$$

$$M_z = 4 - 1,33z \sin \alpha$$

$$M_z = 4 - 0,80z$$

**CORTANTE**

$$Q_z = \frac{dM_z}{dz} ; \quad Q_z = -0,80$$

**NORMAL**

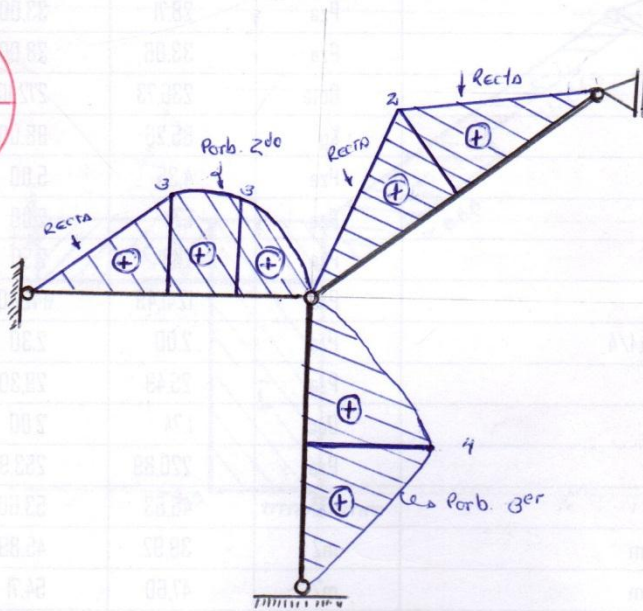
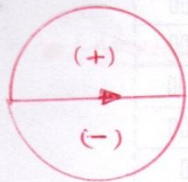
$$N_z = -z \sin \alpha - 1,333 \cos \alpha + z \sin \alpha$$

$$N_z = -1,066$$

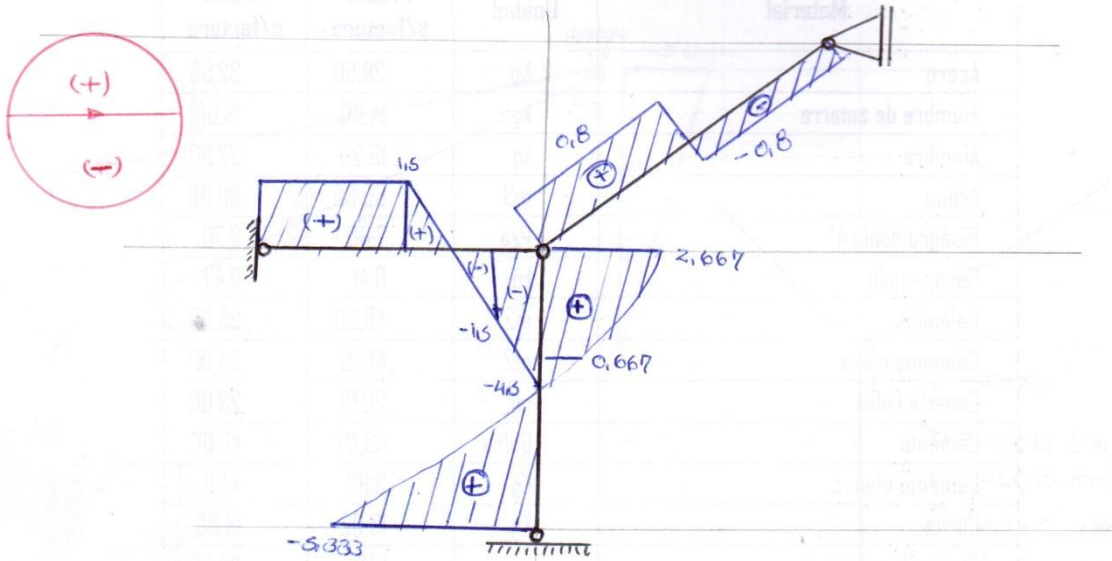
z	M <sub>z</sub>	Q <sub>z</sub>	N <sub>z</sub>
2,50	2,000	-0,80	-1,066
5,00	0,000	-0,80	-1,066

**VI DIAGRAMAS**

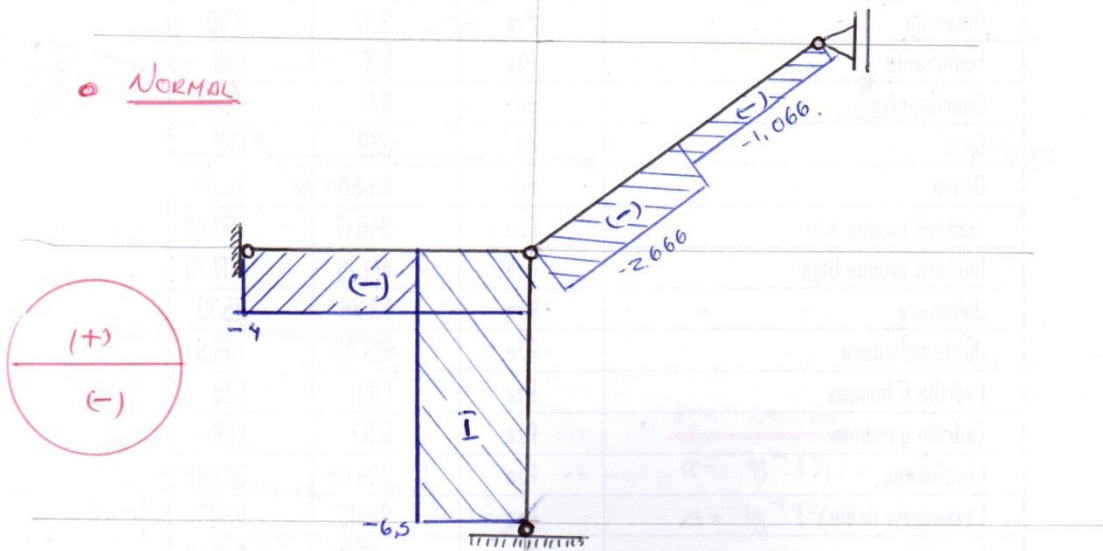
**MOMENTOS**



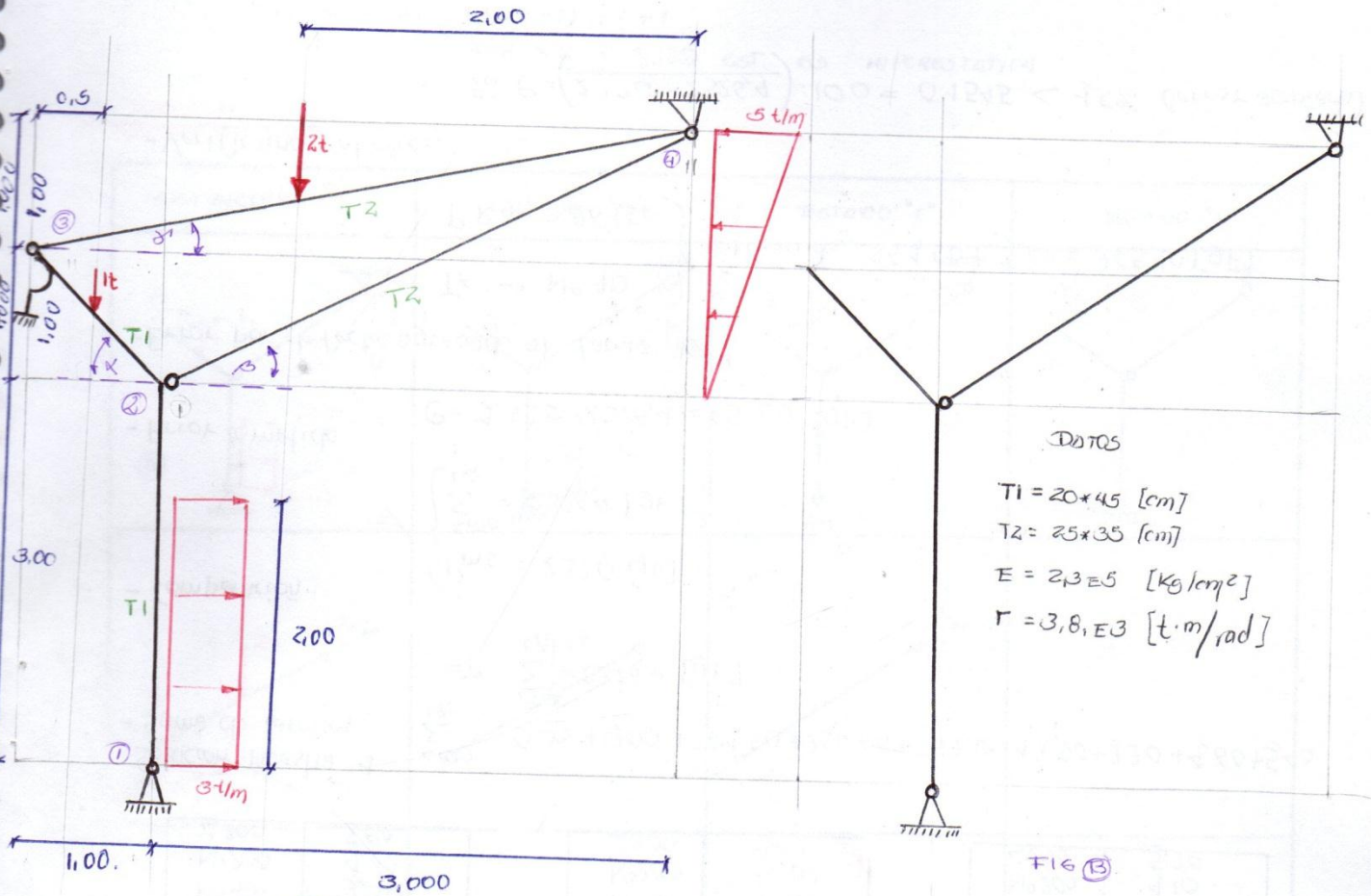
• CORTANTE



• NORMAL



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DATOS  
 $T1 = 20 \times 45$  [cm]  
 $T2 = 25 \times 35$  [cm]  
 $E = 2,3 \text{ E}5$  [kg/cm<sup>2</sup>]  
 $r = 3,8, \text{ E}3$  [t·m/rad]

FIG A

FIG B

SOLUCIÓN

I: CARACTERÍSTICAS GEOMÉTRICAS

I.1 LONGITUDES

$$L_{12} = 3 \text{ [m]}$$

$$L_{23} = \sqrt{1^2 + 1^2} = 1,414 \text{ [m]}$$

$$L_{24} = \sqrt{3^2 + 2^2} = 3,606 \text{ [m]}$$

$$L_{34} = \sqrt{4^2 + 1^2} = 4,123 \text{ [m]}$$

I.2 ANCHURAS

$$\alpha = \text{tg}^{-1}(1/1) = 45^\circ$$

$$\beta = \text{tg}^{-1}(2/3) = 33,69^\circ$$

$$\gamma = \text{tg}^{-1}(1/4) = 14,036^\circ$$

I.3 PRODUCTO EI

$$EI_1 = \frac{0,20 \cdot 0,45^3}{12} \cdot 2,3 \text{ E}6 = 3493,125 \text{ t·m}^2$$

$$EI_2 = \frac{0,25 \cdot 0,35^3}{12} \cdot 2,3 \text{ E}6 = 2054,427 \text{ t·m}^2$$

I.4 PRODUCTO EA

$$EA_2 = 2,3 \text{ E}6 \cdot 0,25 \cdot 0,35 = 201250$$

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II GRADO HIPERESTÁTICO

$$GH = 3(1) - (1+1+1) + 1 + 1$$

$$GH = 2 \quad \downarrow \quad \text{NO EST. ES HIPERESTÁTICA}$$

III ISOSTATIZANDO.

ESTRUCTURA	ESTADO "0"	ESTADO "1"	ESTADO "2"
<p>(A)</p>			
<p>(B)</p>			
<p>(C)</p>			

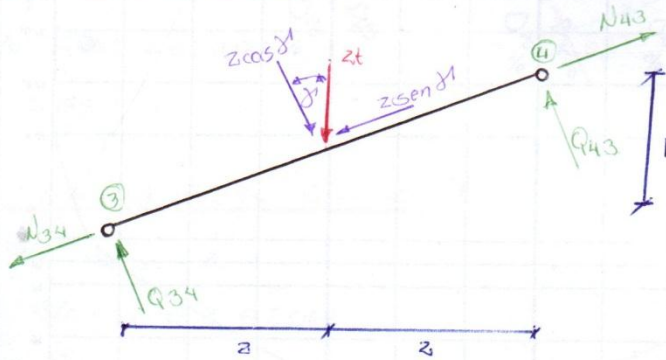
IV. ECUACIÓN DE FLEXIBILIDAD

$$\begin{Bmatrix} D_{10}^A + D_{10}^B + D_{10}^C \\ D_{20}^A + D_{20}^B + D_{20}^C \end{Bmatrix} = \begin{Bmatrix} f_{11}^A + f_{11}^B + f_{11}^C \\ f_{21}^A + f_{21}^B + f_{21}^C \end{Bmatrix} \quad \begin{Bmatrix} f_{12}^A + f_{12}^B + f_{12}^C \\ f_{22}^A + f_{22}^B + f_{22}^C \end{Bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$$

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$$-\begin{Bmatrix} D_{10}^A + D_{10}^B \\ D_{20}^A \end{Bmatrix} = \begin{Bmatrix} f_{11}^A + f_{11}^B & f_{12}^A \\ f_{21}^A & f_{22}^A + f_{22}^B \end{Bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$$

ESTRUCTURA B (BIARTICULADA)



\* CORTANTE

$$\sum M_B = 0 \quad (\uparrow)$$

$$Q_{43} \cdot (4,123) - 2(2) = 0$$

$$Q_{43} = 0,970 \quad \uparrow \quad [\uparrow \eta]$$

$$\uparrow + \sum FQ = 0$$

$$Q_{34} + Q_{43} - 2 \cos \theta = 0$$

$$Q_{34} = 2 \cos 14,036^\circ - 0,970$$

$$Q_{34} = 0,970 \quad \downarrow \quad \uparrow \eta$$

\* NORMALES

$$\sum FH = 0 \quad \rightarrow$$

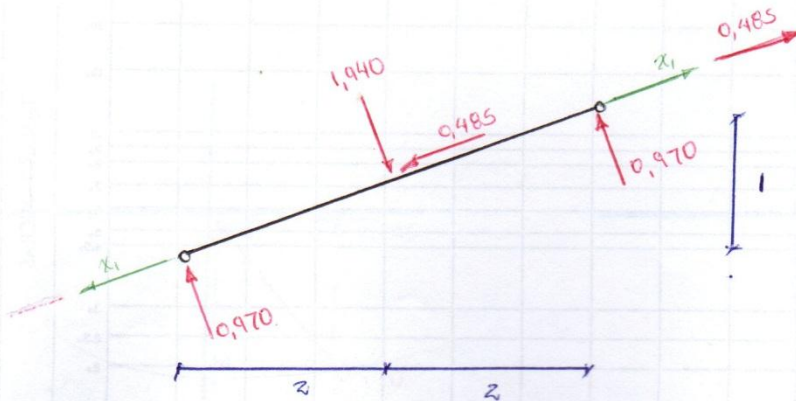
$$-N_{34} - 2 \sin \theta + N_{43} = 0$$

$$N_{43} = N_{34} + 2 \sin \theta$$

$$N_{43} = N_{34} + 0,485$$

$$\text{Si } N_{34} = X_1$$

$$N_{43} = X_1 + 0,485 \quad \downarrow \quad \uparrow \eta$$



Ecuaciones de Normales en el Estado '0'

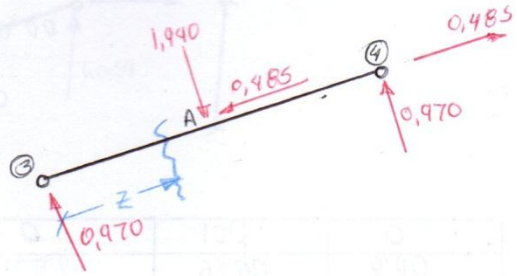
- BARRA. 3/4. ORIG. (3)

⊕ TRAMO 3/A.  $0 \leq z \leq 2,062$

$$N_z = 0 \quad \uparrow \quad \text{tn}$$

⊕ TRAMO. A/4.  $2,062 \leq z \leq 4,123$

$$N_z = 0,485 \quad \uparrow \quad \text{tn}$$



Ecuación de Normales en el Estado '1'

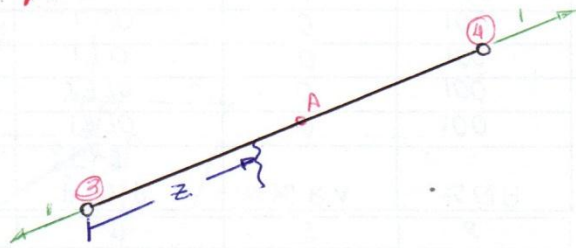
- BARRA. 3/4. ORIG. 3.

⊕ TRAMO. 3/A.  $0 \leq z \leq 2,062$

$$N_z = 1 \quad \uparrow \quad \text{tn}$$

⊕ TRAMO. A/4.  $2,062 \leq z \leq 4,123$

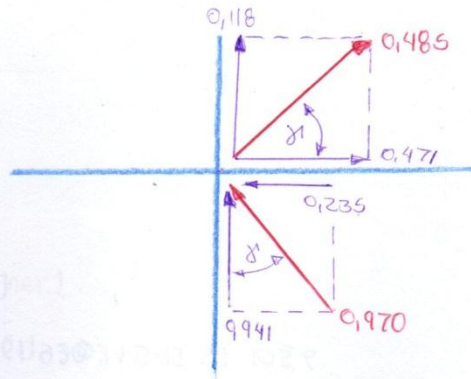
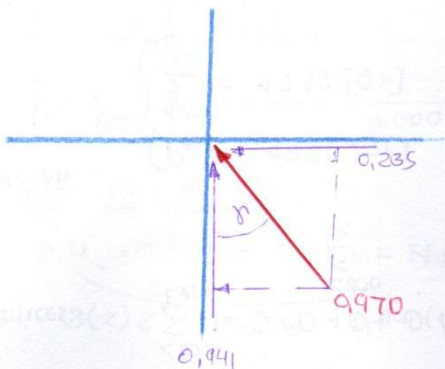
$$N_z = 1 \quad \uparrow \quad \text{tn}$$



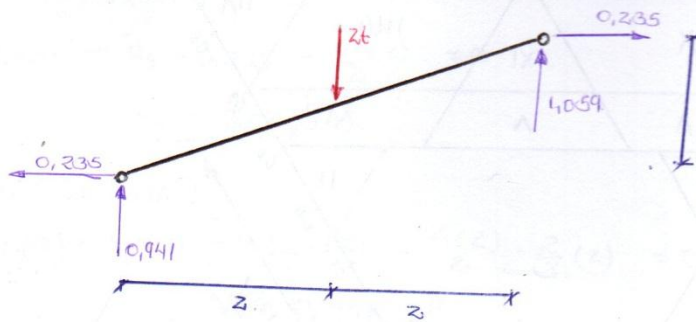
Cálculo de la Deformación y la Flexibilidad en la Estructura. 'B'

$$D_{10}^B = \int_0^{2,062} \frac{0}{EA} dz + \int_{2,062}^{4,123} \frac{0,485(1)}{201250} dz = -4,969 \text{ m}$$

$$f_{11}^B = \int_0^{2,062} \frac{(1)(1)}{201250} dz + \int_{2,062}^{4,123} \frac{(1)(1)}{201250} dz = 2,049 \text{ m/tn}$$

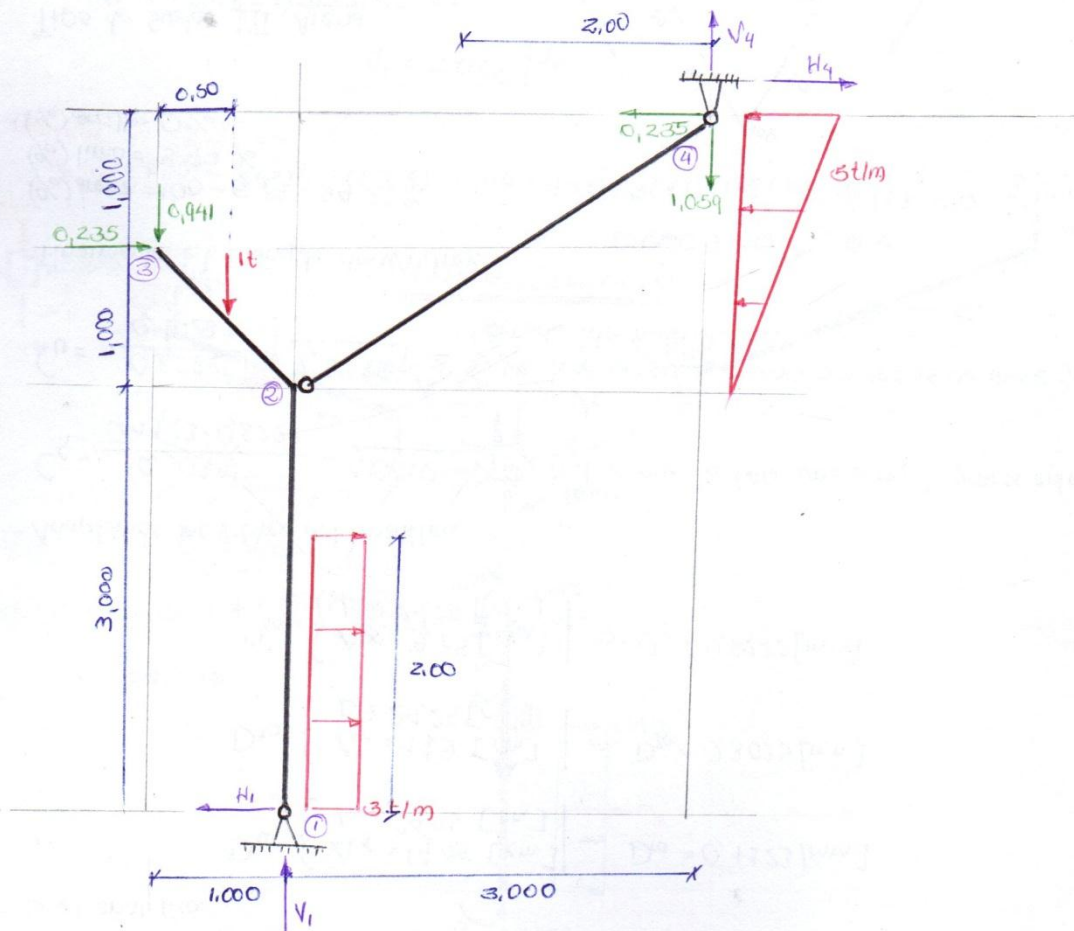


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ESTRUCTURA A

ESTADO "0"



① REACCIONES DE APOYO

a)  $\sum M_2 = 0 \quad (+)$  (A LA IZQ. DE LA ART)

$$H_1 \cdot (3) - 3(2) \cdot 2 - 1(0,500) - 0,941(1) + 0,235(1) = 0$$

$$H_1 = 4,402 \quad \uparrow +$$

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b)  $\sum F_H = 0 \rightarrow$

$$-H_1 + 3(2) + 0,235 + H_4 - 0,235 - \frac{5}{2}(2) = 0$$

$$H_4 = 3,402 \text{ [tn]}$$

c)  $(\sum M_2 = 0 \text{ (A.L. OER A.E.T.)})$

$$H_4 \cdot 2 + 1,059(3) - V_4(3) - 0,235(2) - \frac{5(2)}{2} \cdot \frac{2}{3}(2) = 0$$

$$V_4 = 0,948 \text{ [t]}$$

d)  $\sum F_V = 0$

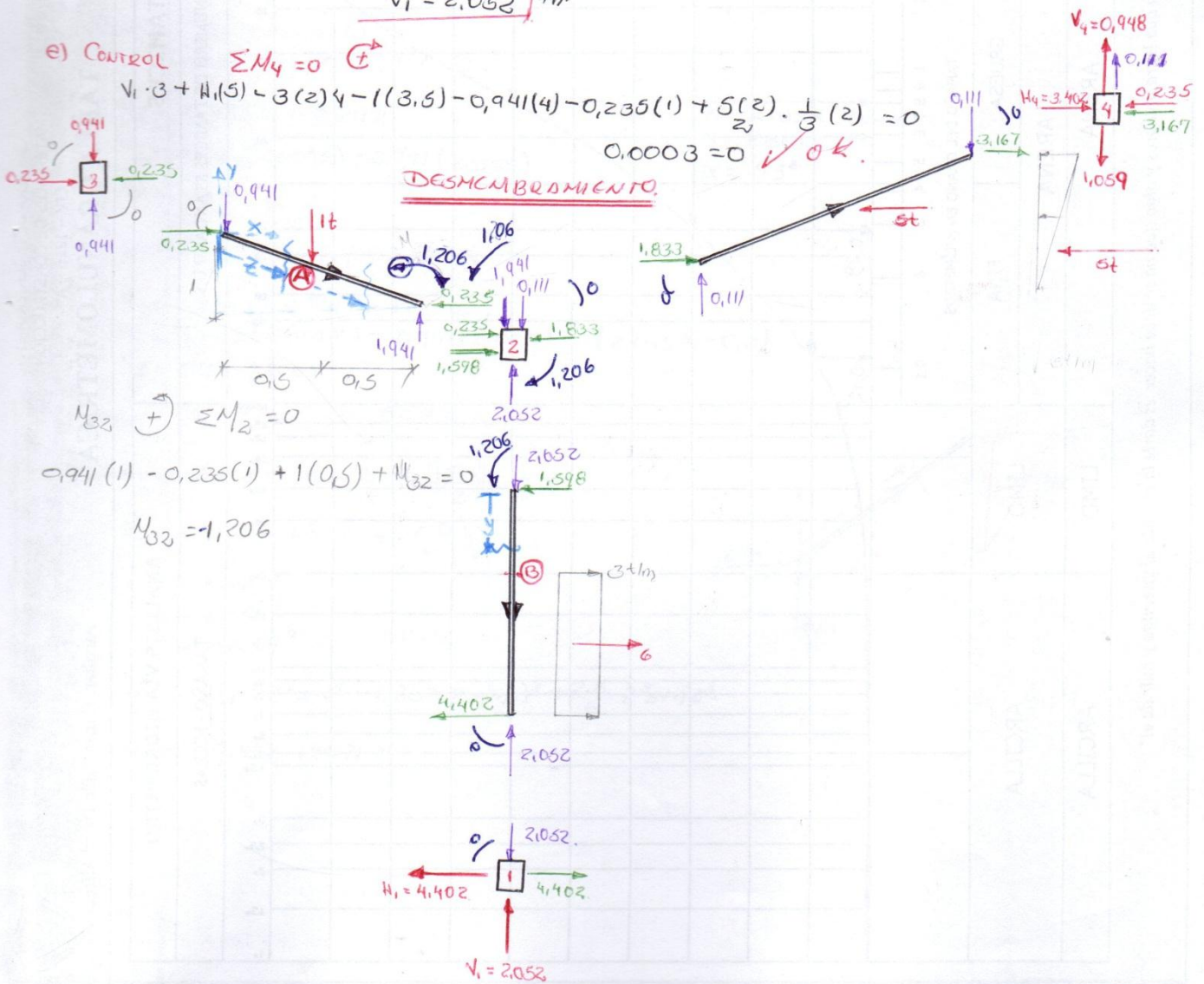
$$V_1 - 0,941 - 0,941 + V_4 = 0$$

$$V_1 = 2,052 \text{ [tn]}$$

e) CONTROL  $\sum M_4 = 0 \text{ (}\oplus\text{)}$

$$V_1 \cdot 3 + H_1(5) - 3(2) \cdot 4 - 1(3,5) - 0,941(4) - 0,235(1) + \frac{5(2)}{2} \cdot \frac{1}{3}(2) = 0$$

$$0,0003 = 0 \quad \checkmark \text{ OK}$$



$M_{32} \oplus \sum M_2 = 0$

$$0,941(1) - 0,235(1) + 1(0,5) + M_{32} = 0$$

$$M_{32} = -1,206$$

③ Ecuación de Momentos.

BARRA 1/2 ORIG. ②

- TRAMO 2/3  $0 \leq y \leq 1$

$$M_y = -1,206 - 1,598 y$$

- TRAMO 0/1  $1 \leq y \leq 3$

$$M_y = -1,206 - 1,598 y + 3(y-1)\frac{(y-1)}{2}$$

$$M_y = -1,206 - 1,598 y + 1,5(y^2 - 2y + 1)$$

$$M_y = 1,5y^2 - 4,598y + 0,294$$

BARRA 2/3 ORIGEN. ③

- TRAMO 3/4  $0 \leq z \leq 0,707$

$$M_z = 0,235 y - 0,941 x$$

$$M_z = 0,235(z \operatorname{sen} \alpha) - 0,941(z \operatorname{cos} \alpha)$$

$$M_z = -0,499 z$$

- TRAMO 1/2  $0,707 \leq z \leq 1,414$

$$M_z = 0,235 y - 0,941 x - 1(x - 0,5)$$

$$M_z = 0,235(z \operatorname{sen} \alpha) - 0,941(z \operatorname{cos} \alpha) - 1(z \operatorname{cos} \alpha - 0,5)$$

$$M_z = 0,5 - 1,206 z$$

BARRA 3/4 ORIG. ②  $0 \leq z \leq 3,606$

$$M_z = 0,111 x - 1,833 y + \frac{9z \cdot y}{2} \cdot \frac{1}{3} y$$

$$M_z = 0,111 x - 1,833 y + \frac{2,5 \cdot y^3}{6}$$

$$x = z \operatorname{cos} \alpha$$

$$y = z \operatorname{sen} \alpha$$

$$\alpha = 33,69^\circ$$

$$M_z = 0,111 z \operatorname{cos} \alpha - 1,833 z \operatorname{sen} \alpha + \frac{2,5}{6} z^3 \operatorname{sen}^3 \alpha$$

$$M_z = -0,92 z + 0,071 \cdot z^3$$

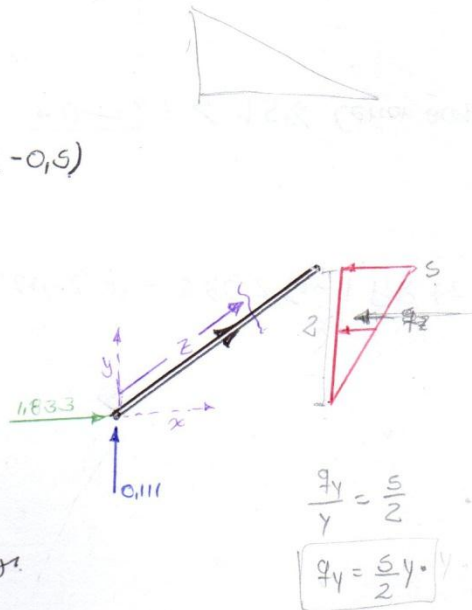
z	Mz
0	-1,206
1	-2,804

z	Mz
1	-2,804
3	0

$$x = z \operatorname{cos} \alpha$$

$$y = z \operatorname{sen} \alpha$$

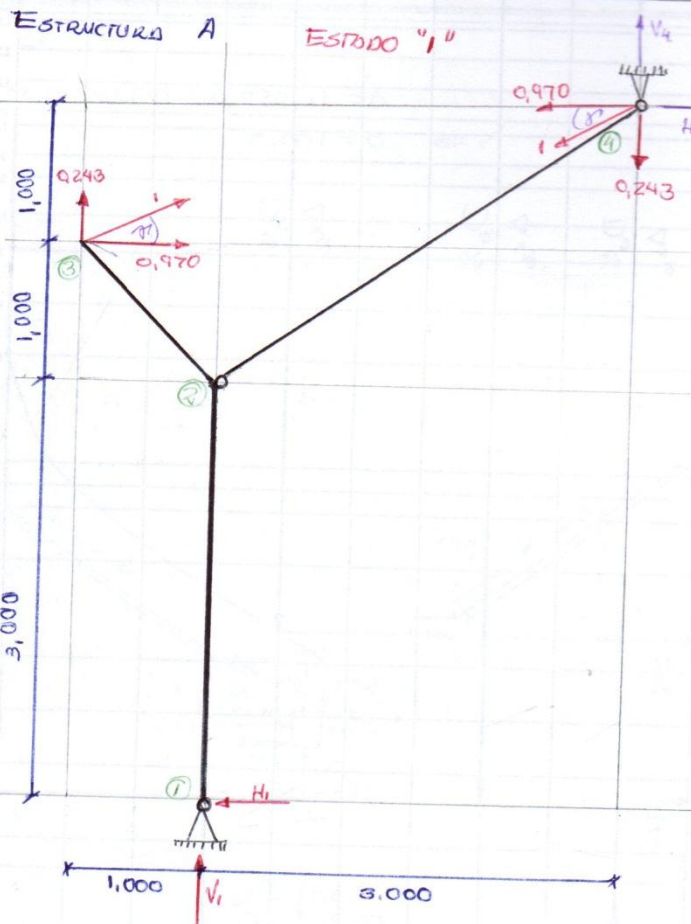
$$\alpha = 45^\circ$$



$$\frac{9y}{y} = \frac{5}{2}$$

$$9y = \frac{5 \cdot y}{2}$$

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1. REACCIONES DE APOYO

a)  $\sum M_3 = 0 \quad (\uparrow)$  (A LA IZQ. DE LO ART.)

$$H_1(3) + 0.243(1) + 0.970(1) = 0$$

$$H_1 = -0.404 \text{ tn}$$

b)  $\sum F_H = 0 \quad (\rightarrow)$

$$-H_1 + 0.970 - 0.970 + H_4 = 0$$

$$H_4 = -0.404 \text{ tn}$$

c)  $\sum M_2 = 0 \quad (\uparrow)$  (A LA DER. ART.)

$$0.243(3) - 0.970(2) + H_4(2) - V_4(3) = 0$$

$$V_4 = -0.673 \text{ tn}$$

d)  $\sum F_V = 0 \quad (\uparrow)$

$$V_1 + 0.243 - 0.243 + V_4 = 0$$

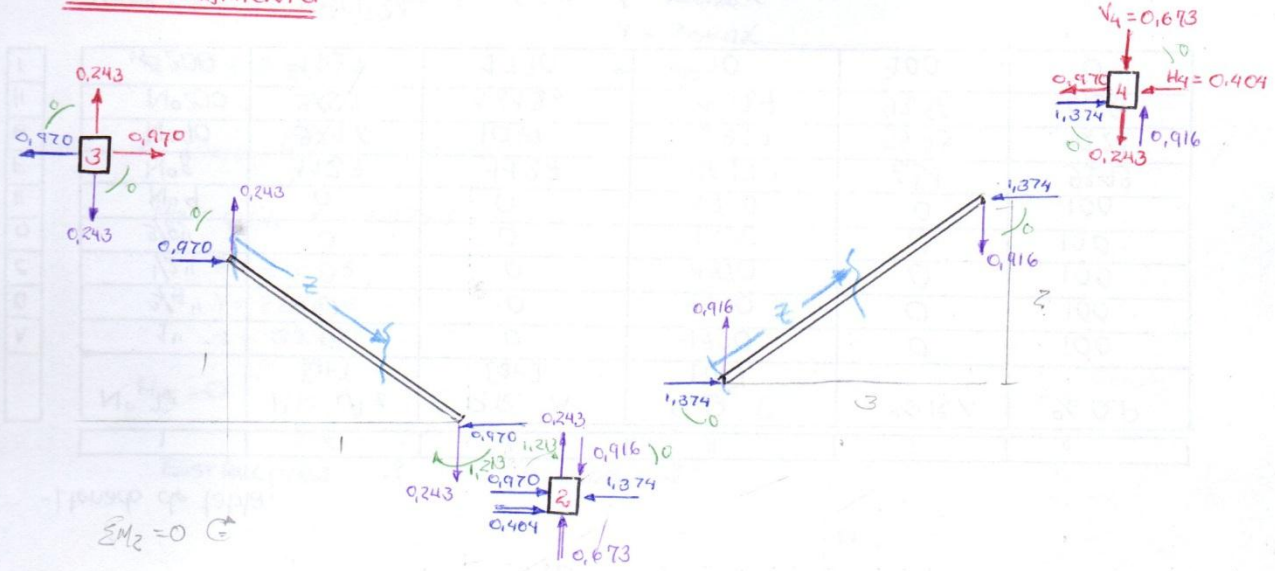
$$V_1 = 0.673 \text{ tn}$$

e) CONTROL  $\Sigma M_4 = 0$   $\oplus$

$$H_1(5) + V_1(3) + 0.243(4) - 0.970(1) = 0 \quad ? = 100$$

$$0.001 = 0 \quad \text{OK} \checkmark$$

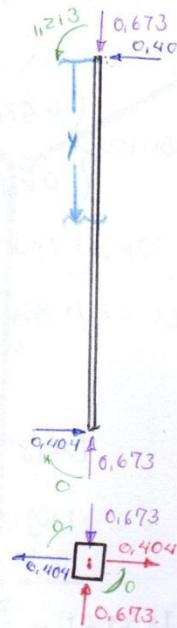
② DESMEMBRAMIENTO



$\Sigma M_2 = 0$   $\oplus$

$$-M_2 + 0.970(1) + 0.243(1) = 0$$

$$M_2 = 1.213$$





3: ECUACIONES DE MOMENTOS.

Barra 1/3 ORIG. ②  $0 \leq y \leq 3.$

$$M_y = -0.404y + 1.212$$

Barra. 2/3 ORIG. ③  $0 \leq z \leq 1.414.$

$$M_z = 0.243x + 0.970y$$

$$M_z = 0.058z.$$

$$x = z \cos \alpha$$

$$y = z \sin \alpha$$

$$\alpha = 45^\circ.$$

Barra. 3/4 ORIG. ④  $0 \leq z \leq 3.606$

$$M_z = 0.916x - 1.374y$$

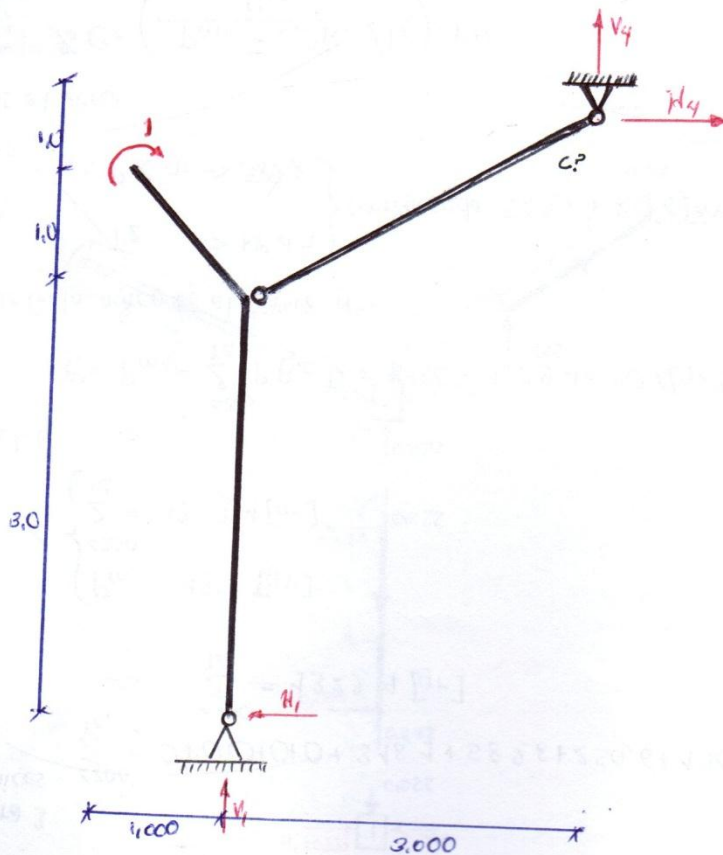
$$x = z \cos \beta$$

$$y = z \sin \beta$$

$$\beta = 33.69^\circ$$

$$M_z = 0$$

ESTRUCTURA A.  $E = 7000 \frac{kg}{cm^2}$



1: REACCIONES DE APOYO.

a)  $\sum M_2 = 0 \quad (\uparrow)$  (A LO IZQ. DE LO ART.)

$H_1(3) + 1 = 0$

$H_1 = -0.333 \text{ (tn)}$

b)  $\sum F_H = 0$

$-H_1 + H_4 = 0$

$H_4 = -0.333 \text{ [tn]}$

c)  $\sum M_2 = 0 \quad (\uparrow)$  (A LO DER. DE LO ART.)

$H_4(2) - V_4(3) = 0$

$V_4 = -0.222 \cdot \text{tn}$

d)  $\sum F_V = 0$

$V_1 + V_4 = 0$

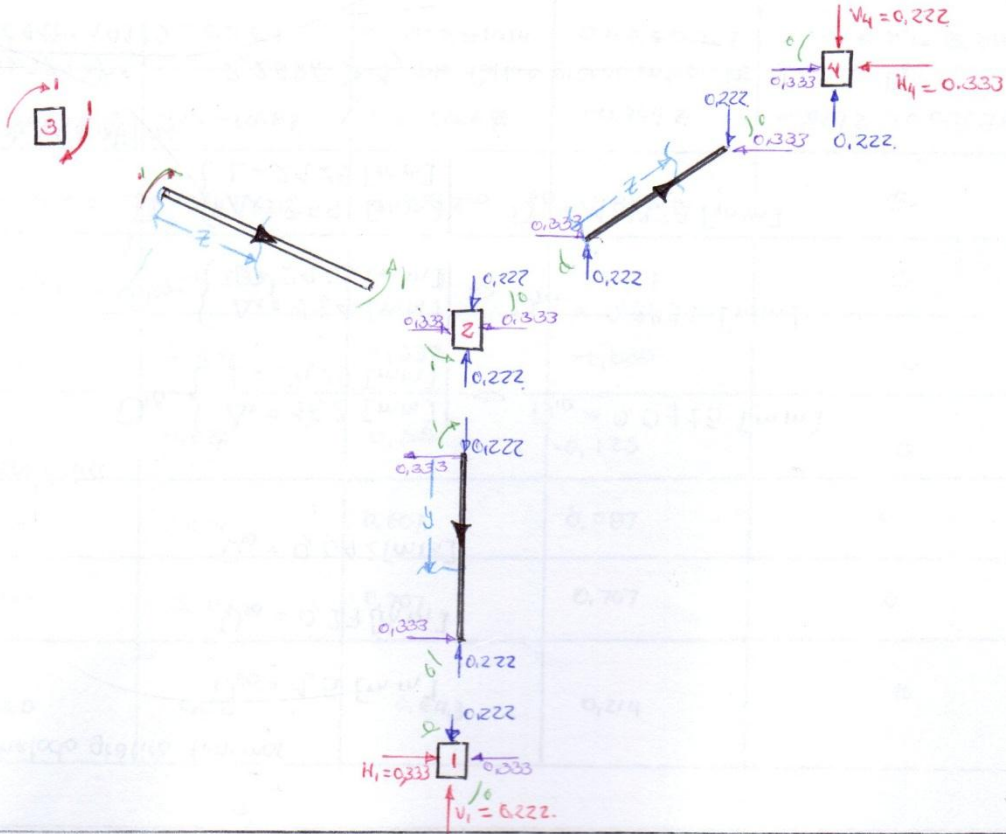
$V_1 = 0.222 \text{ tn}$

e) CONTROL.  $\sum M_4 = 0 \quad (\uparrow)$

$H_1(5) + V_1(3) + 1 = 0$

$0,001 = 0 \quad \text{OK.}$

2: DESMONTAJE ENTO:



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3; ECUACIÓN DE MOMENTOS

Barra 1/2 ORIG. (2)  $0 \leq y \leq 3$

$$M_y = 1 - 0,333y$$

Barra 2/3 ORIG. (3)  $0 \leq z \leq 1,414$

$$M_z = 1$$

Barra 2/4 ORIG. (4)  $0 \leq z \leq 3,606$

$$M_z = 0,222x - 0,333y$$

$$x = z \cos \beta$$

$$y = z \sin \beta$$

$$\beta = 33,69^\circ$$

$$M_z = 0$$

Barra	(1)	(2)	(3)	(4)	(5)
EI	3493,126		3493,126		2054,427
OR.	(2)		(3)		(2)
LIMITES	$1 \leq y \leq 3$	$0 \leq y \leq 1$	$0,707 \leq z \leq 1,414$	$0 \leq z \leq 0,707$	$0 \leq z \leq 3,606$
$N_0$	$1,5y^2 - 4,598y + 0,294$	$-1,206 - 1,598y$	$0,5 - 1,206z$	$-0,499z$	$-0,924z + 0,071z^3$
$N_1$	$-0,404y + 1,212$	$-0,404y + 1,212$	$0,858z$	$0,858z$	0
$N_2$	$1 - 0,333y$	$1 - 0,333y$	1	1	0
$\int N_0 M_1$	-2,318	-1,971	-0,532	-0,050	0
$\int N_0 M_2$	-1,916	-1,627	0,551	-0,125	0
$\int N_1 M_1$	0,435	1,034	0,607	0,087	0
$\int N_2 M_2$	0,297	0,704	0,707	0,707	0
$\int M_1 M_2$	0,360	0,853	0,643	0,214	0

CÁLCULO DE DEFORMACIONES Y FLEXIBILIDADES

$$D_{10}^A = \frac{-2,318 - 1,971 - 0,532 - 0,050}{3493,125} = -4,394 \text{ E-3}$$

$$D_{20}^A = \frac{-1,916 - 1,627 - 0,551 - 0,125}{3493,125} = -1,208 \text{ E-3}$$

$$f_{11}^A = \frac{0,435 + 1,034 + 0,607 + 0,087}{3493,125} = 6,192 \text{ E-4}$$

$$f_{22}^A = \frac{0,297 + 0,704 + 0,707 + 0,707}{3493,125} = 6,914 \text{ E-4}$$

$$f_{21}^A = f_{12}^A = \frac{0,360 + 0,853 + 0,643 + 0,214}{3493,125} = 5,926 \text{ E-4}$$

HALANDO LAS REDUNDANTES

$$\begin{Bmatrix} -1,394 \text{ E-3} + 4,969 \text{ E-6} \\ -1,208 \text{ E-3} \end{Bmatrix} = \begin{Bmatrix} 6,192 \text{ E-4} + 2,049 \text{ E-5} \\ 5,926 \text{ E-4} \end{Bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} + \begin{Bmatrix} 5,926 \text{ E-4} \\ 6,914 \text{ E-4} + \frac{1}{3,8 \text{ E}3} \end{Bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}$$

Resumen

$$X_1 = 2,351$$

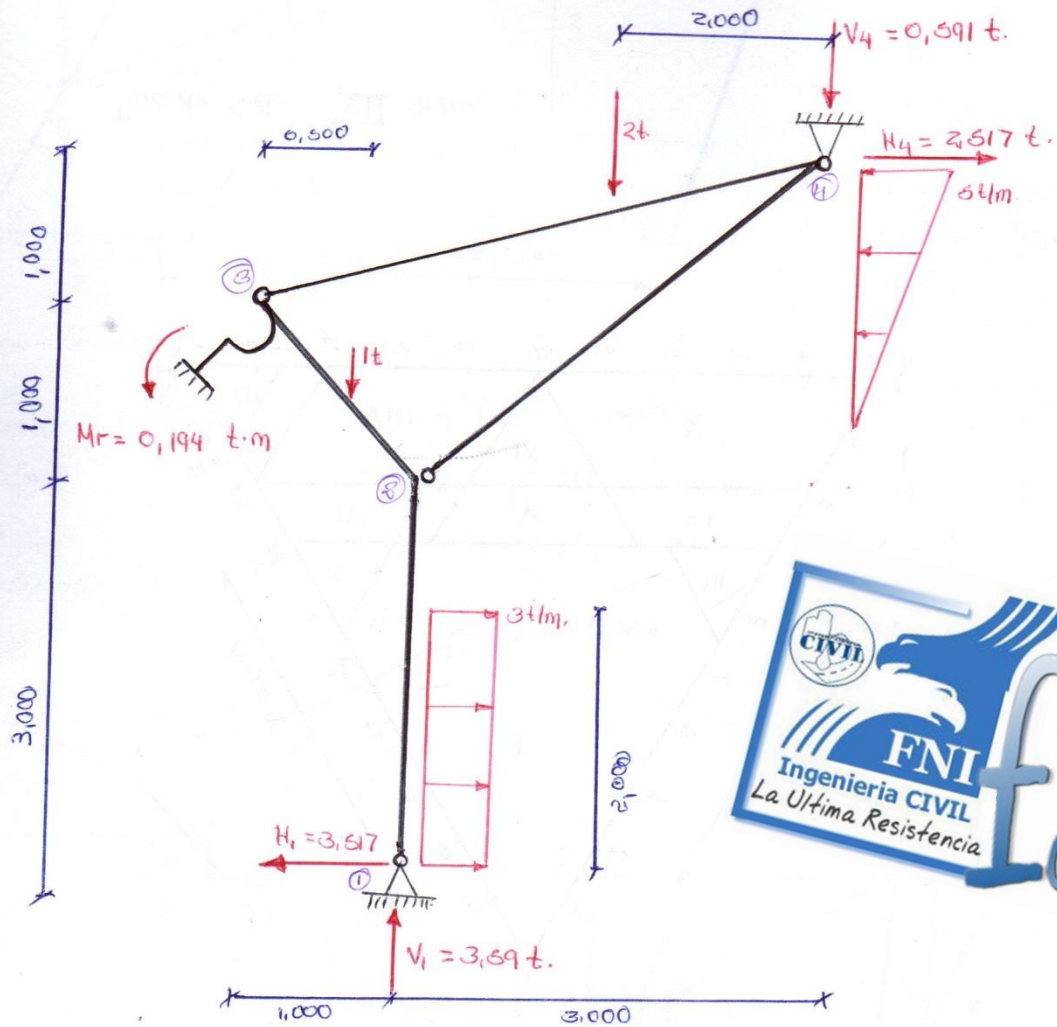
$$X_2 = -0,194$$

HALANDO LAS REACCIONES FINALES

$$\begin{Bmatrix} H_1 \\ V_1 \\ H_4 \\ V_4 \end{Bmatrix} = \begin{Bmatrix} 4,402 \\ 2,052 \\ 3,402 \\ 0,948 \end{Bmatrix} + (2,351) \star \begin{Bmatrix} -0,404 \\ 0,673 \\ -0,404 \\ -0,673 \end{Bmatrix} + (-0,194) \star \begin{Bmatrix} -0,333 \\ 0,222 \\ -0,333 \\ -0,222 \end{Bmatrix} = \begin{Bmatrix} 3,517 \\ 3,591 \\ 2,517 \\ -0,591 \end{Bmatrix}$$

EST "0"                      EST "1"                      EST "2"

LAS REACCIONES SON:



VERIFICACIÓN

$$\rightarrow \sum F_H = 0 \quad -H_1 + 6 + H_4 - 5 = 3,517 + 6 + 2,517 - 5 = 0 \quad \text{OK. } \checkmark$$

$$0 = 0$$

$$\uparrow \sum F_V = 0 \quad V_1 - 1 - 2 - V_4 = 3,591 - 1 - 2 - 0,591 = 0 \quad \text{OK. } \checkmark$$

$$0 = 0$$

$$\curvearrow \sum M_O = 0 \quad -0,194 \cdot 6(1) - 1(0,5) + 2(1) + V_4(3) + H_4(5 \cdot 3 + 4/3) = 0$$

$$0,001 = 0 \quad \text{OK. } \checkmark$$